

# MECHANICAL FAULT DIAGNOSIS BASED ON FUZZY CLUSTERING ALGORITHM

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**Abstract:** To improve the accuracy of mechanical fault diagnosis, based on the full vector spectrum analysis and fuzzy clustering algorithm, two kinds of improved FCM algorithms were proposed, namely vector -DKFCM (fuzzy kernel clustering based on density function) and vector -UGAFCM (uniform genetic fuzzy clustering) intelligent fault diagnosis method. Meanwhile, the new mathematical model and its concrete realization flow were given, and the effectiveness of the new method was verified. Moreover, because the cluster validity index is domain correlation, the validity index for classification of rotating machinery common fault was compared and analyzed. The experimental method was and the results showed that three fuzzy clustering validity indexes VXB, VKwon and VCWB applicable to the field of rotating machinery fault diagnosis were obtained. In summary, the fuzzy clustering algorithm, compared with traditional method, has higher classification accuracy.

**Keywords:** fuzzy clustering; full vector spectrum; fault diagnosis; validity.

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## 1. Introduction

In recent years, rotating machinery is becoming more and more large-scale, intelligent and complex. Then, the condition monitoring and fault diagnosis technology for these devices is becoming more and more important. The process of fault diagnosis is essentially the process of pattern recognition for various types of faults [1].

Based on homologous information fusion, the full vector spectrum analysis technology is used to fuse multi-channel data, which can correctly reflect the real motion of the rotor, and is compatible with the traditional spectral analysis. At the same time, the numerical algorithm is simple, robust and fast, and is more conducive to intelligent fault diagnosis [2,3].

Fuzzy clustering algorithm is a kind of pattern recognition. As one of the most classical one in these algorithms - fuzzy C mean (FCM) algorithm, it has more and more applications in fault diagnosis because of its unsupervised and large sample size [4].

The fuzzy C mean algorithm assumes that the data set is ellipsoid, so the algorithm is not ideal for non-hyper sphere data, noise contaminated data and asymmetric data. In order to adapt to different data structure types, people put forward various fuzzy clustering algorithms. For instance, the introduction of kernel method, mountain function, potential function and density function into cluster analysis achieves the purpose of improving and perfecting algorithm [5,6].

However, because FCM is very sensitive to data structure and initial value, it is difficult to get the best clustering.

Therefore, based on full vector spectrum analysis, two improved algorithms are put forward and applied to fault recognition of rotating machinery.

## 2. Methodology

### 2.1 Plane full vector spectrum analysis technique

The analysis method combining the fusion technology of homologous multi-sensor information and the study of the dynamic characteristics of rotating machinery and the mechanism of rotation is the full information analysis method.

There are three kinds of full information analysis technologies widely used at present: holographic spectroscopy, full spectrum technology and all vector spectrum technology. Compared with the other two methods, the full vector spectrum has many special advantages, which is more conducive to the intelligent fault diagnosis [7,8].

The vibration signals obtained by sensors in two different directions in a stationary state are in the plane of the probe with a number of harmonic  $\omega_k(k=1,2,\dots,n)$  combined steady vorticity.

For any harmonic k, we set:

$$\begin{cases} x_{ck} = X_k \cos \phi_{xk} \\ x_{sk} = X_k \sin \phi_{xk} \\ y_{ck} = Y_k \cos \phi_{yk} \\ y_{sk} = Y_k \sin \phi_{yk} \end{cases} \quad (1)$$

$X_k$  and  $Y_k$  are the amplitude of signal measured by harmonic k in x and y directions;  $\phi_{xk}$  and  $\phi_{yk}$  are the phase of signal measured by harmonic k in x and y directions. In engineering practice, both  $x_k$  and  $y_k$  are real numbers. The true motion relationship of the rotor node is shown in Figure 1.

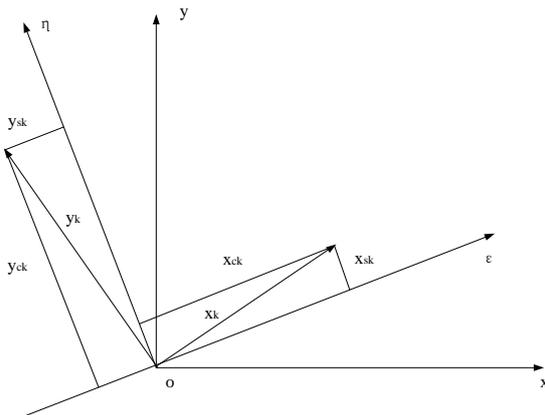


Figure 1 The actual motion at the rotor junctions

The motion trajectory of the signal is ellipse under the harmonics, and Figure 2 is a schematic diagram of its motion trajectory. Full vector spectrum has the following definition: elliptic motion trajectory of rotating machinery under single harmonic, the semi major axis is the main vibration vector under harmonic set to  $R_L$ , short semi axis is the assistant vibration vector under harmonic set to  $R_S$ , the angle between the main vibration vector and the x axis is  $\alpha$  and the phase angle of axis along the elliptic track is  $\phi$ .

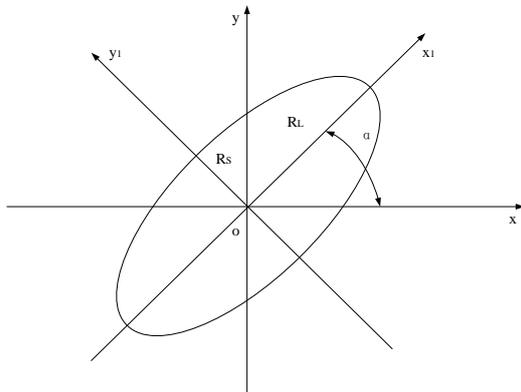


Figure 2. Motion trajectory of harmonic signal

In the kinematics, ellipse motion trajectory can be decomposed into two eccentric motions with the same frequency but the opposite motion direction.

The relationship between the ellipse parameters and two eccentric parameters is shown below:

$$\begin{cases} R_{LK} = X_{pk} + X_{rk} \\ R_{SK} = X_{pk} - X_{rk} \\ \phi_{ak} = \phi_{pk} \\ 2\alpha_k = \phi_{pk} + \phi_{rk} \end{cases} \quad (2)$$

When  $X_{pk} > X_{rk}$ , the synthesis trajectory under k harmonic and eccentric motion with  $X_{pk}$  as the radius have the same motion direction. At this time, we call the rotor in a positive motion state; when  $X_{pk} < X_{rk}$ , the synthesis trajectory has the same motion direction as eccentric motion with  $X_{rk}$  as the radius; when  $X_{pk} = X_{rk}$ , the synthetic trajectory is a straight line. By (2), it can be seen that, for elliptic trajectory under any harmonic frequency, there are: the sum of two circle radius is the main vibration vector of the trajectory; the difference between two circle radius is the assistant vibration vector of the trajectory; the initial phase of elliptic trajectory is the same as that of positive precession circle; the initial phase of positive precession circle is equal to 2 times the angle between the trajectory and the axis.

On the flexible rotor test platform, the X and Y channels are sampled synchronously and periodically with eddy current sensors, and the vibration signals of X and Y channels under the condition of supporting looseness are collected. The rotor speed is 2870rpm, and the sampling frequency is 2048.5HZ. The X and Y channel signals are fused by full vector spectrum and their spectra are shown in Figure 3.

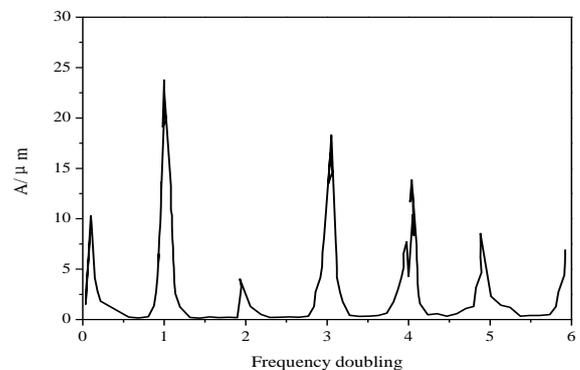


Figure 3. Full vector spectrum fusions

The traditional spectral analysis takes the single channel signal in the same section as the research object, which neglects the connection between the two channel signals, and does not guarantee the integrity of the information. As a result, it greatly affects the reliability and accuracy of the fault diagnosis. However, the full vector spectrum analysis makes up for this deficiency and fundamentally solves the problem [9]. From the full vector spectrum in Figure 3, we can see that the amplitude of its fundamental frequency is obviously larger than its characteristic frequency, and the fractional harmonic amplitude is also larger. Besides, the

frequency components are also more consistent with the characteristics of the supporting looseness, which makes it easier to make judgement. From the above analysis, we can see that the full vector spectrum analysis technology can ensure the integrity of information because of the integration of dual channel information. In consequence, it can reflect the fault characteristics more truly and comprehensively, which also improves the accuracy of fault diagnosis.

### 2.2 Fuzzy clustering algorithm

Many concepts in fault diagnosis are fuzzy, such as strong vibration and unstable vibration. Considering the difficulty and the real-time and intelligent requirements of the method in engineering practice, the most widely used field of fault diagnosis is fuzzy clustering method based on objective function [10].

Fuzzy C- means algorithm is the most popular kind of fuzzy clustering algorithm. In a given sample set X, the cluster number c and the weight m, we can

$$\begin{cases} J_{km} = \sum_{i=1}^c \sum_{k=1}^n \mu_{ik}^m \|\Phi(x_k) - \Phi(v_i)\|^2 & m \in [1, \infty) \\ s.t. U \in M_{fc} \end{cases} \quad (3)$$

In the above formula,  $\Phi()$  suggests nonlinear mapping from the mode space to high dimensional feature space.  $\Phi(x_k)$  refers to the image of sample  $x_k$  in the corresponding kernel space and  $\Phi(v_i)$  represents the image of clustering prototype  $v_i$  in corresponding kernel space.

No matter it is the classical FCM algorithm or the kernel function KFCM algorithm, they are ultimately reduced to the constrained optimization problem. Therefore, the same as most nonlinear optimization problems, the clustering result is affected by the initial value. The randomly given initial cluster prototype is the most commonly used method, but the result is that the probability for the clustering result to obtain the global optimal solution is small. In this paper, a relatively superior density function method is used to initialize the cluster prototype.

$$D_k^{(i)} = D_k^{(i-1)} - D_i^* \frac{1}{1 + f_d \|x_k - x_i^*\|^2} \quad (i=1, 2, \dots, c-1) \quad (4)$$

c is a cluster number,  $D_i^* = \max\{D^{(i-1)}_k, k=1, 2, \dots, n\}$ , and the corresponding  $x_i^*$  takes i-th cluster centre. The fuzzy kernel clustering algorithm introducing density function is based on KFCM algorithm. It uses c clustering prototype obtained by density function method to replace a set of clustering prototypes

obtain the best fuzzy membership matrix and clustering prototype through repeated iteration. The specific steps are as follows: initialize and give the number of clusters c and weight m, the maximum number of iterations T, iterative stop threshold  $\epsilon$  and clustering prototype  $V^{(0)}$ ; calculate or update the objective function; obtain or update the membership matrix U; update clustering prototype V; algorithm terminates and outputs membership matrix U and clustering prototype V.

The theory of pattern recognition points out that, for a low dimensional space, a linearly separable pattern can be linearly separable if it is mapped to high-dimensional feature space by some nonlinear. The functions that satisfy the Mercer theorem can be used as a kernel function. Kernel method is an effective way to solve linear inequalities. In addition, kernel-based clustering algorithm solves the problem that classical clustering algorithm is not suitable for many kinds of data [11,12]. If the Euclid distance is used in the high dimensional feature space, the fuzzy clustering objective function of the feature space is as follows:

obtained by random initialization. This can avoid the influence caused by the algorithm of random initialization clustering prototype, and improve the probability of obtaining a satisfactory solution to a certain extent.

Genetic algorithm (GA) with global search performance is combined with FCM to form a new algorithm - genetic algorithm fuzzy clustering method (GAFCM). The algorithm searches the approximate global optimal solution. On this basis, the solution is used as the initial clustering prototype of FCM, and the strong local search function of iterative algorithm is used to strive for the optimal solution. The mathematical model of the basic genetic algorithm is expressed as follows:

$$SGA = (C, E, P_0, M, \Phi, \Gamma, \Psi, T) \quad (5)$$

C suggests encoding method of individual; E indicates the fitness function;  $P_0$  means the initial population; M represents the population size;  $\Phi$  is a selection algorithm;  $\Gamma$  denotes the crossover algorithm;  $\Psi$  shows the variation algorithm; T refers to the calculation termination condition. The flow chart of the GA operation using the basic genetic operator is shown in Figure 4.

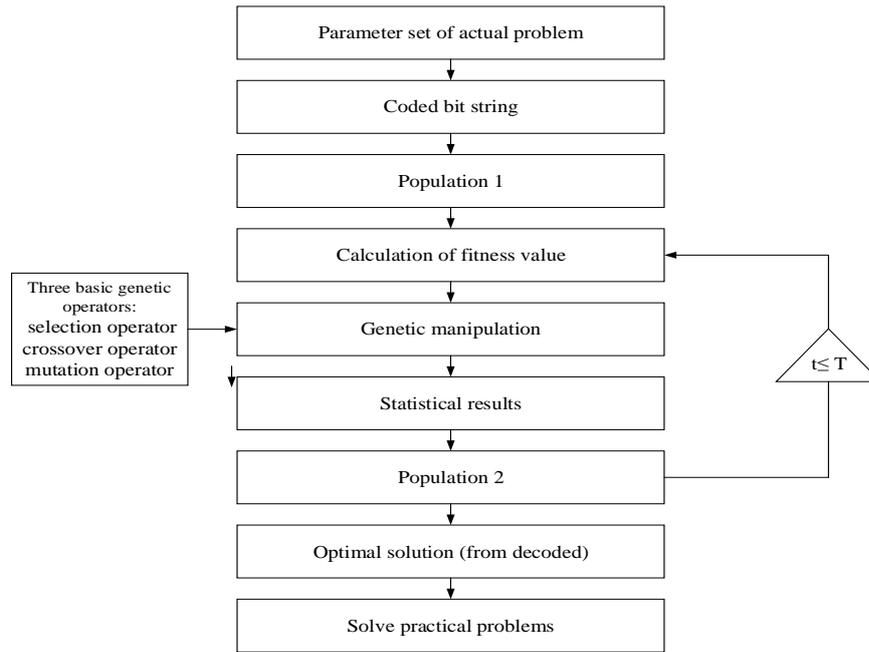


Figure 4. Genetic algorithm flow chart

There are three basic operators in genetic operator: selection, crossover and mutation operator.

The following is the common features: the operations of operators on population are carried out under random perturbation; the operation effect is influenced by a variety of factors, such as

population size, operator operation probability, encoding scheme, initial population and fitness function setting; operators' strategies are determined with regard of specific problem solution. The three commonly used genetic operators are shown in Figure 5.

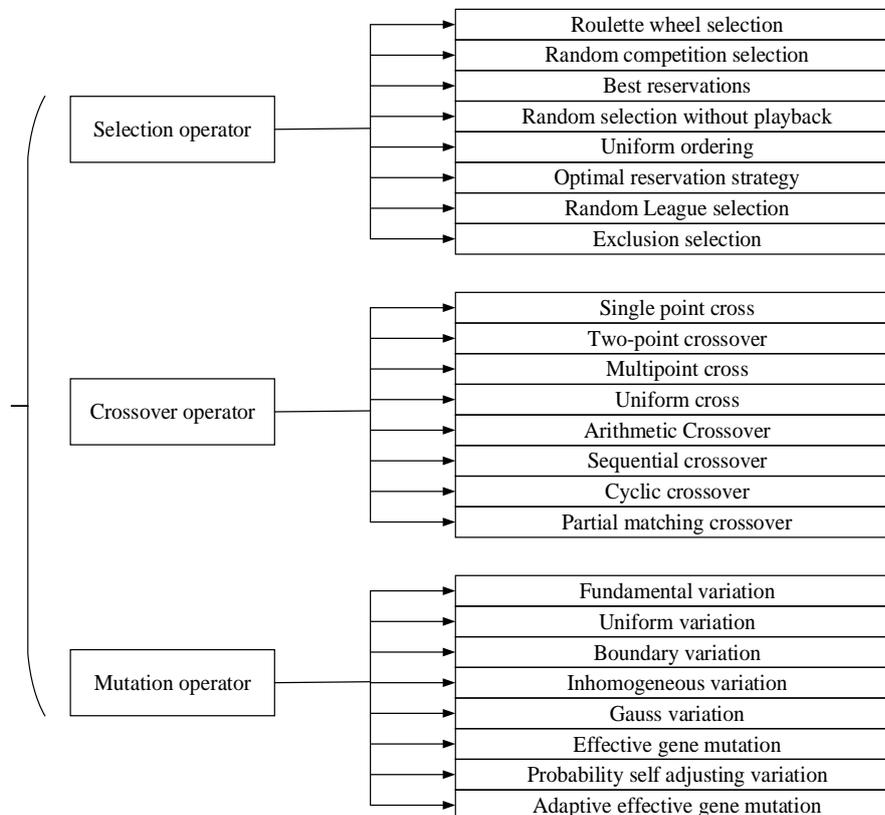


Figure 5. Common genetic operators

Although the mutation operator in GA has a local search function, a large number of studies have shown that the local optimization function of GA is still poor. The GA algorithm good at global optimization and FCM good at local search are combined to form a new GAFCM algorithm. The advantages of them are used to obtain the optimal

solution of search space. First of all, we use GA algorithm to get the approximate global optimal solution, and then use the optimal solution as the initial clustering centre of FCM algorithm. Finally, we use FCM algorithm for iterative search to get the optimal solution. The concrete steps are as follows.

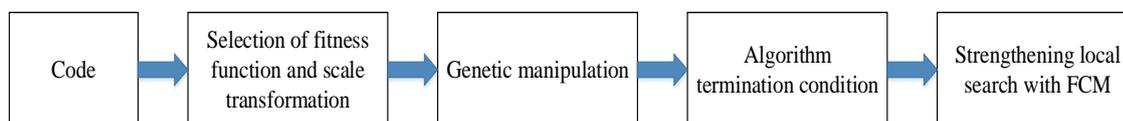


Figure 6. The steps of genetic fuzzy algorithm

### 2.3 Clustering validity analysis

As we all know, clustering analysis belongs to the category of unsupervised learning. There is no prior knowledge for the structure of data sets needed to be clustered. Therefore, no matter what method is used, the reasonableness of the results needs to be evaluated. At present, the validity of the clustering is measured by using the index of clustering validity function.

For clustering analysis based on objective functions, the clustering validity can usually be transformed into the problem of determining the

best number of clustering  $c$ . The clustering validity function can be used to determine the optimal number of clusters through the iterative process shown in Figure 7. Generally speaking, clustering validity can be measured from two angles: one is the inner class compactness, that is, the pattern in the same class has high similarity; the other is the inter class separation degree, that is, the patterns in different classes are different as far as possible. The existing clustering validity functions can be roughly divided into three categories: fuzzy partition based on data set, partition based on data set structure and statistical information based on +data set.

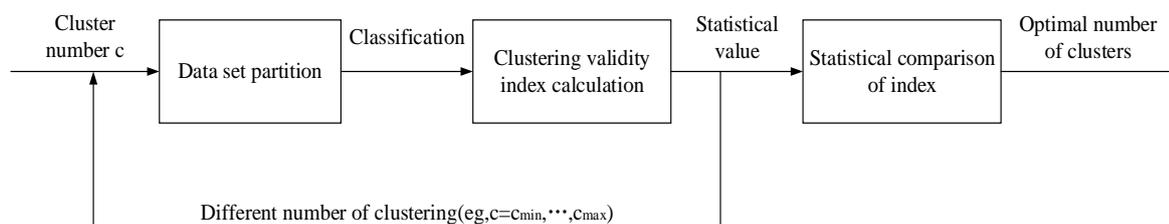


Figure 7. Determine the best cluster number flow chart

The effectiveness index commonly used in fuzzy clustering is shown in Table 1, where  $c$  is the number of clusters,  $U$  is the membership matrix,  $V$  is the cluster centre,  $n$  is the sample number, and  $m$  is the fuzzy control index.

Table 1 Fuzzy clustering validity index

Nr.	Indexes
1	Partition coefficient ( $V_{PC}$ )
2	Partition entropy ( $V_{PE}$ )
3	Improved partition coefficient ( $V_{MPC}$ )
4	Xie-Beni effectiveness index ( $V_{XB}$ )
5	S.H.Kwon effectiveness index ( $V_{Kwon}$ )
6	$V_{FS}$ index ( $V_{FS}$ )
7	A.M.Bensaid effectiveness index ( $V_{bsaid}$ )
8	$V_{CWB}$ index ( $V_{CWB}$ )
9	$V_{WSJ}$ index ( $V_{WSJ}$ )
10	$V_{COS}$ index ( $V_{COS}$ )

Although the indexes  $V_{XB}$ ,  $V_{Kwon}$ ,  $V_{FS}$ ,  $V_{bsaid}$ ,  $V_{CWB}$  and  $V_{WSJ}$  are different in form, they all contain compactness index and separation index, while the combination of them is different. The  $V_{COS}$  index adds the degree of overlap based on this. The above indexes are widely used in engineering practice.

## 3. Results and discussion

### 3.1 Application of full vector - fuzzy kernel clustering in fault diagnosis of rotating machinery

Fault feature extraction is done by full vector spectrum analysis and initial clustering prototype is obtained by density function method. Fuzzy classifier is built based on kernel function to form fault classifier, and a new fault diagnosis method based on full vector - fuzzy kernel clustering is formed. In view of the advantages of the full vector spectrum, density function method and fuzzy kernel

clustering method, it is expected to get good application in the field of mechanical fault diagnosis.

They are combined and applied to the fault identification of the rotating machinery. The

concrete steps are as follows and the diagnostic flow chart is shown in Figure 8.

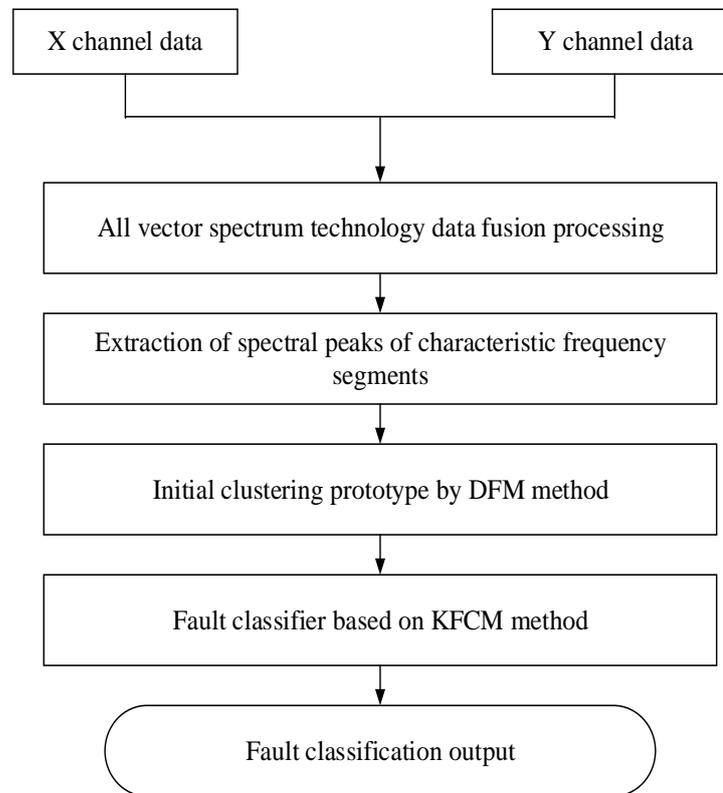


Figure 8. Diagnostic flowchart

For the feature extraction of signals, the amplitude of c, y channels and the main vibration vectors in 9 bands of 0.01f~0.39f, 0.40f~0.49f, 0.50f, 0.51f~0.99f, 1.0f, 2.0f, 3.0f, 4.0f and 5.0f are extracted as the feature vectors of samples, respectively.

The newly proposed method is tested.

The kernel function used in DKFCM algorithm is commonly used radial basis function in engineering. The nuclear parameter  $\sigma^2$  is 1.7, and the test results are shown in Figure 9 and Figure 10.

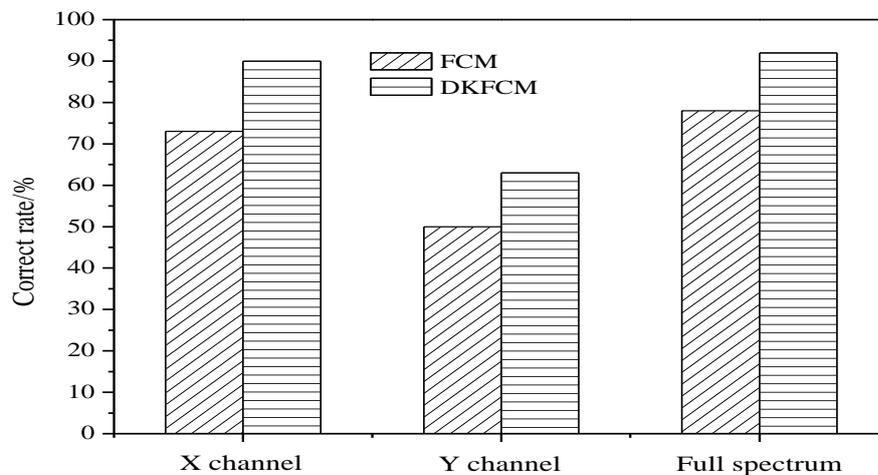


Figure 9. Comparison of correct rate of fault recognition

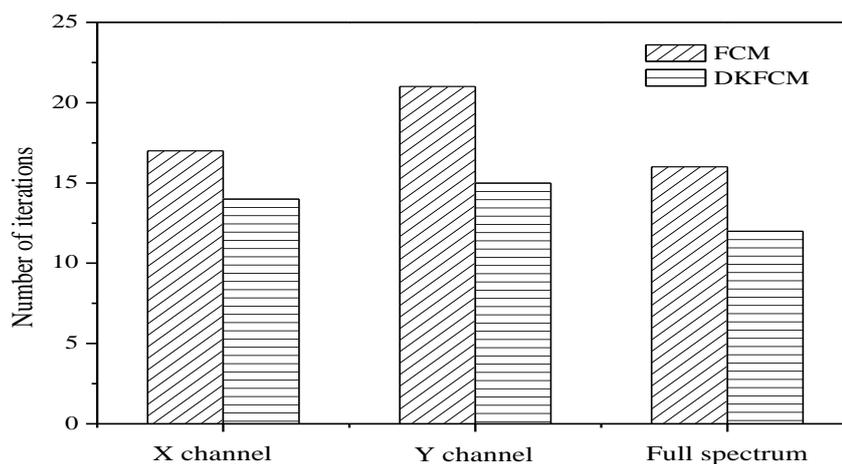


Figure 10. Iteration number comparison

From the fault recognition results in graph, we can draw the following conclusions: both the classical FCM algorithm and a new DKFCM algorithm, the correct rate of classification based on X channel signal for feature extraction and classification based on Y channel signal for feature extraction (single channel signal feature extraction) is lower than that of the classification based on full vector spectrum analysis for feature extraction. No matter what way (single channel or the way based on full vector spectrum) of feature extraction is adopted, the correct rate of classification of the new DKFCM algorithm is higher than that of the classical FCM algorithm. The number of iterations based on the new DKFCM algorithm is less than that based on the classical FCM classification.

The final function of pattern recognition in fault diagnosis is to predict the unknown fault. Therefore, in order to further verify the engineering application of the newly proposed DKFCM algorithm, the algorithm is used to classify the fault category for

samples to be tested. The steps of full vector - DKFCM method applied to fault prediction are as follows [13]: apply the full vector spectrum technology for the feature extraction of typical fault samples and predicted samples; adopt DKFCM method clustering for typical fault samples and obtain the clustering centre; place the clustering centre obtained and the predicted samples for clustering, get the membership matrix of each sample, and judge the category of predicted samples according to the maximum membership principle.

10 samples in four working conditions that can be correctly classified are taken as training samples and training models. 4 predicted samples are taken as follows: the sample 1 is the support loosening fault; the sample 2 is the rubbing fault; the sample 3 is the normal state; the sample 4 is the unbalanced fault. The clustering centre and the sample to be measured are used as the input samples of the KFCM algorithm to cluster again. The fuzzy membership matrix is obtained as shown in Table 2.

Table 2 Fuzzy membership matrix

Out-off-balance	Rub	Loose	Normal	Sample 1	Sample 2	Sample 3	Sample 4
0.0002	0.0002	0.8961	0.0000	0.9336	0.0001	0.0000	0.0002
0.0006	0.9993	0.0359	0.0000	0.0225	0.9991	0.0000	0.0005
0.0004	0.0005	0.0420	1.0000	0.0266	0.0006	0.9998	0.0004
0.9986	0.0001	0.0260	0.0000	0.0170	0.0001	0.0000	0.9988

According to the maximum membership degree principle, it is clear from Table 2 that the four prediction samples have obtained the correct classification results. This further validates the effectiveness of new method.

### 3.2 Full vector - uniform genetic fuzzy clustering fault diagnosis method

Uniform design is an application of the method of number theory, which belongs to the category of

"pseudo Mon Castro method". The method is based on orthogonal design, but it abandons the requirement of "neat and comparable". It only requires the "uniform dispersion" of test points, which largely reduces the number of trials. The way to achieve uniform design is to use a set of carefully designed tables, including the design table and the corresponding use table. It is introduced into the algorithm, and the genetic fuzzy clustering algorithm is optimized by the uniform design method. The UGAFCM algorithm is shown in Figure 11.

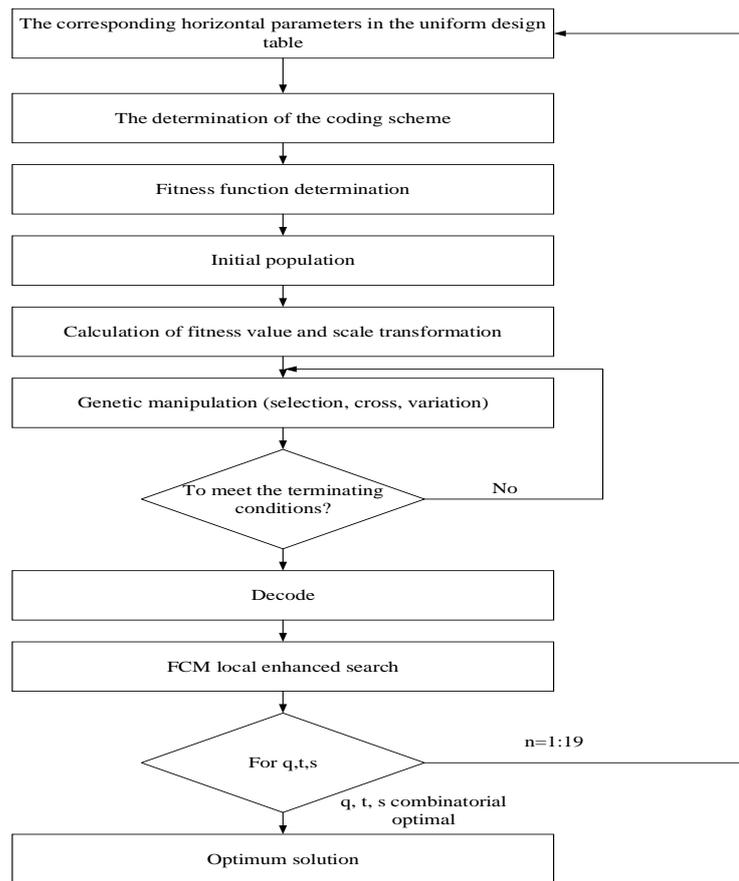


Figure 11. UGAFCM algorithm flowchart

The flow chart of full vector uniform genetic fuzzy clustering method for fault diagnosis is shown in Figure 12. The steps are as follows: use the full vector spectrum method as feature extraction tool; apply UGAFCM algorithm to establish fault classifier;

recognize fault, pass feature vector of samples to be identified from fault classifier, obtain the fuzzy membership matrix according to samples to be identified and judge the fault category of samples based on the maximum membership principle.

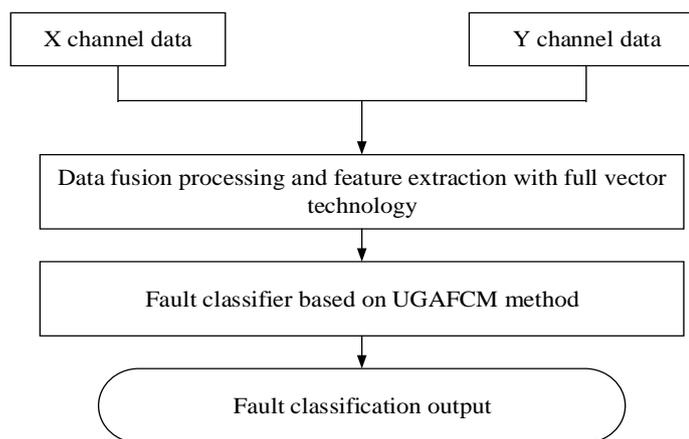


Figure 12. Flow chart of fault diagnosis for full vector -UGAFCM method

3 typical faults commonly seen in the rotor system of Bently flexible rotor test rig are: imbalance, rubbing and support loosening. 25 sets of data are collected synchronously to X and Y two channels in 3 fault states and normal conditions. The sampling frequency is 2048.5HZ, the motor speed is 2400rpm,

the population size of the GAFCM algorithm is  $M = 140$ , the cross probability  $P_c = 0.88$ , and the fuzzy control index is  $m = 1.89$  [14]. In order to verify the effectiveness of the new method, the new method is compared with the traditional method, and the results are shown in Figure 13.

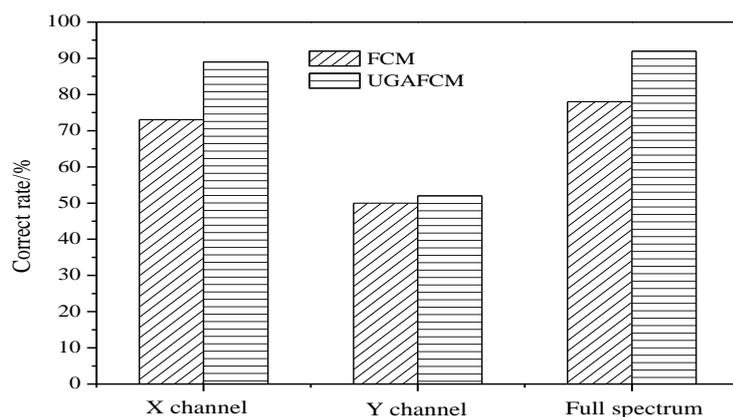


Figure 13. Comparison of fault identification results

It can be seen from the figure that, whether it is the traditional FCM algorithm or the new UGAFCM algorithm, the fault classification correct rate based on x or y channel signal is lower than that of the full vector spectrum accuracy for feature extraction. Whether it is the feature extraction based on x channel and Y channel or full vector spectrum, the classification correct rate of new UGAFCM algorithm is significantly higher than that of the traditional FCM algorithm.

### 3.3 Fuzzy clustering validity evaluation

DKFCM method and UGAFCM method are used to verify the validity of clustering effectiveness function in the field of rotating machinery fault diagnosis. The DKFCM method and the GAFKM method are used to test the validity of the three indicators. The best number of clusters is known to be 4. The experimental results are shown in Figure 14, Figure 15 and Figure 16.

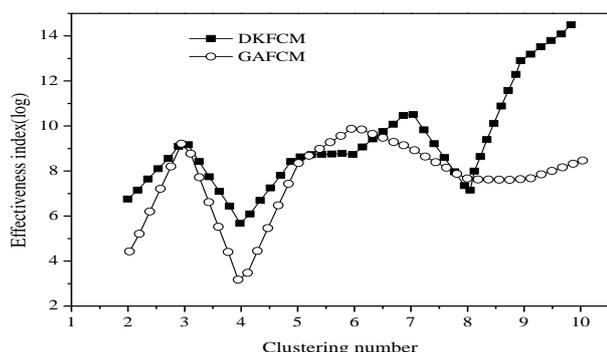


Figure 14 The effectiveness of  $V_{XB}$  index

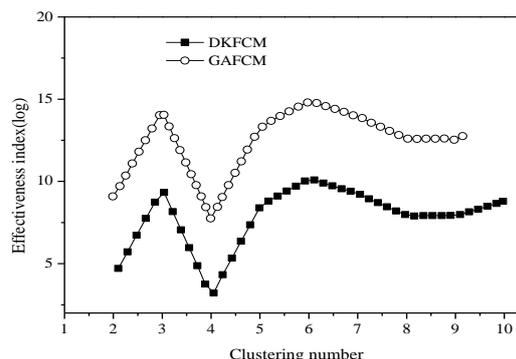


Figure 15 The effectiveness of  $V_{Kwon}$  index

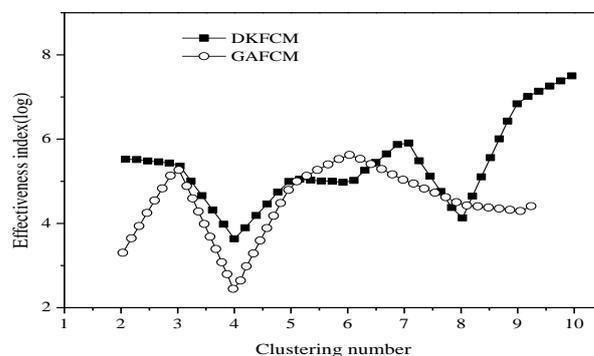


Figure 16 The effectiveness of  $V_{CWB}$  index

It can be seen from the figure that, no matter it is classified by using DKFCM algorithm or GAFKM algorithm, the number of clustering of three indexes  $V_{XB}$ ,  $V_{Kwon}$  and  $V_{CWB}$  in the best classification is 4.

For two different kinds of clustering algorithms, the change trend of clustering validity index is almost the same.

## 4. Conclusion

According to the fuzzy C clustering shortcomings, the kernel function method and density function method were introduced, DKFCM algorithm was proposed to improve it and the full vector spectrum and the DKFCM algorithm were combined and applied in fault diagnosis of rotating machinery. In view of the complementary feature of genetic algorithm and FCM, the two were combined. Based on applying GA in searching approximate global optimal solution, FCM algorithm was used for the local optimal search to obtain the optimal value of the solution space. At the same time, the uniform design method was adopted to search better parameter combination. Full vector spectrum technology and GAFCM were combined for fault recognition of rotating machine, and the validity of the above two improved methods was verified by experiments. It proved that it had higher classification accuracy than traditional method.

In this study, feature extraction method, kernel function selection, kernel parameter setting, combined parameter setting and genetic operators' selection greatly affect the performance and accuracy of fault diagnosis. Therefore, how to select the relevant parameters suitable for the fault diagnosis field and how to reduce the running time of the algorithm will be the focus of the later research.

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