

# DYNAMIC RESPONSES COUPLED ANALYSIS OF FPSO IN SOUTH CHINA SEA

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**Abstract** - The far-flung ocean has abundant resource and the exploitation area of ocean oil and gas is developing from offshore to deepwater and ultra-deepwater following the increase demand for oil and gas resource. Therefore, one of the floating structures for deepwater exploitation is FPSO. In this article, the coupling hydrodynamic problems of FPSO and mooring system have been studied and analyzed by using the three dimensional potential theory, the FPSO's mooring system is simulated by slender rod theory. In the 1500m deepwater, calculated and analyzed the hydrodynamic and motion of six-Degree of Freedom (6DOF) of FPSO and mooring lines' load of FPSO at the fairlead. Then compared the result of our program with software Ariane by BV, it can be seen that the result of program conform to the actual needs. Therefore, use the program to calculate the hydrodynamic response of Single Point Mooring (SPM) FPSO in South China Sea with the wind/wave/flow. The results indicate that the sway/roll/yaw of SPM-FPSO are quite well while surge motion is a little large. The static and dynamic analysis of FPSO's mooring system is calculated, when the direction of wave/wind accord with the motion, the force of mooring lines at fairlead reach maximum while the moment is minimum.

**Keywords:** Coupling Hydrodynamic, Mooring System, Static Analysis, Dynamic Analysis

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## 1. Introduction

The far-flung ocean has abundant resource and the exploitation area of ocean oil and gas is developing from offshore to deepwater and ultra-deepwater following the increase demand for oil and gas resource. In terms of situation in China, the South China Sea have abundant resource, the oil and gas exploration and development has great potential. There are four classic offshore are utilized [1], they are TLP (Tension Leg Platform), SPAR, Semi-Submersible Platform and FPSO. FPSO has low cost, short construction period, portable and oil/gas storage capacity, so FPSO received extensive attention of the industry. To solve the hydrodynamic of FPSO is one of the most important problems. The coupling analyze of FPSO and its mooring system, FPSO and its turret.

The FPSO is assumed undergoing six-Degree of Freedom (6DOF) motions under the external environmental loads, such as wind/wave/current forces. The natural frequency of FPSO's surge/sway/yaw is low, FPSO will generate significant movement when effect of wave drift force. Approximation methods for calculating the difference frequency force was presented by Newman [2], in the case of calculate the difference frequency force without second-order velocity

potential. It can be save the calculation time effectively. The motion response analysis of FPSO with turret was presented by Wichers [3-5], but regardless the dynamic effect of the mooring lines and risers to FPSO hull. Huse and Matsumoto [6] propose the dominant mechanisms by through static water attenuation test which shows damping of mooring lines belongs low frequency energy dissipation. Aranha [7, 8] consider the effect of wave drift with flow velocity. Fine and Grue [9] obtain the damping of low frequency yaw by solve the radiation/diffraction problem with low speed. Yang and Teng [10-12] developed a full time domain analysis method to calculate the wave force and the result accurately to second-order. Recently, there are three models were used to simulate mooring line, such as the lumped mass model, the catenary model and slender rod model. Koterayma [13] obtained the result of frequency domain and time domain by lumped mass model. The three-dimensional slender rod theory was presented by Garret [14] to calculate the dynamic performance of the floating structure and mooring system. Kim, Soo, Young [15] to study the nonlinear dynamic response of mooring lines by Newmark method. In order to solve the coupling problem of floating structure and mooring system, mainly adopts the time domain analysis and frequency domain analysis [16-20].

In this paper, we use the three-dimensional potential theory to compute the hydrodynamic coefficients, motion response of 6DOF, then calculate the load of mooring lines at fairlead by use slender rod theory in the dynamic analysis and static analysis, respectively [21-24].

## 2. Hydrodynamic of FPSO and Slender Rod Theory

### • Potential theory

The Cartesian coordinate system  $(x, y, z)$  is established on the static water surface, the  $z$ -axis oriented positively upward. The velocity potential equation can be given according to the ideal fluid assumptions:

$$\Delta\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} = 0 \tag{1}$$

Where the  $\Phi$  is velocity potential.

Nonlinear free-surface condition and wave-surface equation are:

$$\left\{ \frac{\partial^2\Phi}{\partial t^2} + g \frac{\partial\Phi}{\partial z} + 2\nabla\Phi\nabla \frac{\partial^2\Phi}{\partial z^2} = 0 \right\}_{z=\eta} \tag{2}$$

$$\eta = \left\{ -\frac{1}{g} \left( \frac{\partial\Phi}{\partial t} + \frac{1}{2} \nabla\Phi \bullet \nabla\Phi \right) \right\}_{z=\eta} \tag{3}$$

The velocity potential satisfy the boundary conditions of the seabed is:

$$\frac{\partial\Phi}{\partial z} = 0 \quad z = -h \tag{4}$$

The  $h$  indicate the water depth. The velocity potential also satisfy the corresponding to the body-surface condition and the radiation condition. In order to solve the velocity potential and keep to the second-order items by perturbation expansion when the hypothesis of slight wave.

$$\Phi = \varepsilon\phi^{(1)} + \varepsilon^2\phi^{(2)} + O(\varepsilon^3) \tag{5}$$

$$\eta = \varepsilon\eta^{(1)} + \varepsilon^2\eta^{(2)} + O(\varepsilon^3) \tag{6}$$

Where  $\varepsilon = kA$  is perturbation coefficient,  $k$  indicate the wave number,  $A$  is wave amplitude.

To solve the equations (2) and (3) by Taylor series expansion when  $z = 0$ , then substitute the equations (5) and (6) into equation of free-surface, respectively.

$$\eta^{(1)} = -\frac{1}{g} \frac{\partial\phi^{(1)}}{\partial t} \tag{7}$$

$$\eta^{(2)} = -\frac{1}{g} \frac{\partial\phi^{(2)}}{\partial t} - \frac{1}{2g} \nabla\phi^{(1)}\nabla\phi^{(1)} + \frac{1}{g^2} \frac{\partial\phi^{(1)}}{\partial t} \frac{\partial^2\phi^{(1)}}{\partial z\partial t} \tag{8}$$

$$\frac{\partial^2\phi^{(1)}}{\partial t^2} + g \frac{\partial\phi^{(1)}}{\partial z} = 0 \tag{9}$$

$$\frac{\partial^2\phi^{(2)}}{\partial t^2} + g \frac{\partial\phi^{(2)}}{\partial z} = -2\nabla\phi^{(1)}\nabla \frac{\partial\phi^{(1)}}{\partial t} + \frac{1}{g} \frac{\partial\phi^{(1)}}{\partial t} \left( \frac{\partial^3\phi^{(1)}}{\partial t^2\partial z} + g \frac{\partial^2\phi^{(1)}}{\partial t^2} \right) \tag{10}$$

### • Slender rod theory

The slender rod model has strongly versatility, it can be simulate the mooring line and flexible bar such as marine riser, and better computation efficiency and precision.

The equilibrium equation of slender rod section can be provided by Newton's second law and moment equilibrium conditions.

$$q + F' = \rho\ddot{r}(s, t) \tag{11}$$

$$M' + r' \times F + m = 0 \tag{12}$$

Where,  $q$  is uniform external load by per unit,  $F$  is the shear stress on the unit,  $M$  is bending moment on the unit,  $m$  is torque by per unit,  $r'$  indicate the tangent direction of unit vector on the bar,  $r(s, t)$  is the displacement of pole central-line.

Assumption for uniform distribution, the axial stiffness of the total moment as follows:

$$M = r' \times EIr'' + Hr' \tag{13}$$

Where  $EI$  and  $H$  indicate flexural rigidity and torque on the bar, respectively.

Combine the equations (12) and (13) can obtain the equation as follows:

$$r' \times \left[ (EIr'')' + F \right] + H' r' + Hr'' + m = 0 \tag{14}$$

Then dot product  $r'$  for equation (14), that is:

$$H' + mr' = 0 \tag{15}$$

When  $H = 0$ ,  $m = 0$ , the equation (15) can be written as:

$$r' \times \left[ (EIr'')' + F \right] = 0 \tag{16}$$

Post-multiplication  $r'$  for equation (16), can be obtained as:

$$r' \times \left[ (EIr'')' + F \right] \times r' = 0 \tag{17}$$

Assumption the mooring line elongation is:

$$(r' * r') = (1 + \varepsilon)^2 \tag{18}$$

Let,

$$\lambda = r' * \left[ (EIr'')' + F \right] = T - EI\kappa^2 \tag{19}$$

Where,  $F = -(EIr'')' + \lambda r'$ ,  $\kappa$  is the curvature of mooring line,  $T$  indicate the tension of mooring line.

So the governing equation of slender rod is:

$$-(EIr'')'' + (\lambda r')' + q = \rho\ddot{r}(s, t) \tag{20}$$

The equation (20) and (18) are the basic governing equations of mooring line's static analysis and dynamic analysis.

### 3. Results and Discussion

The table 1 shows the main parameters of FPSO in this study. Fig. 1 shows the model of FPSO in this research.

Table 1. The main parameters of FPSO

Lpp (m)	310.0
Breadth at waterline (m)	47.17
Draft (m)	18.90
Vertical CoG above keel (m)	13.32
Longitudinal CoG in front of Lpp/2 (m)	6.83
Displacement (kg)	2.3862E + 08
Roll radius of gyration (m)	14.77
Pitch radius of gyration (m)	77.47
Yaw radius of gyration (m)	79.30
Turret position (in front of Longitudinal CoG) (m)	84.62
Turret elevation below FPSO keel (m)	1.52
Turret diameter (m)	15.85
Front wind area (m <sup>2</sup> )	1012
Transverse wind area (m <sup>2</sup> )	3772

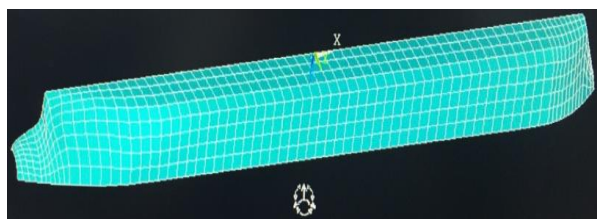


Figure 1. Model of FPSO

• **The analysis of 6DOF motions response of FPSO**

There is six Degree of Freedom motions for FPSO under the external environmental loads, such as surge, sway, heave, roll, pitch and yaw. The table 2 gives us the statistic of 6DOF motions of FPSO. The Mean is short for mean value, Std is short for standard deviation, Max is short for maximum and Min indicate minimum.

Table 2. The statistic of 6DOF motions of FPSO

Motion	Mean	Std	Max	Min
Surge (m)	-0.10062E + 03	0.10083E + 02	-0.68745E + 02	-0.13326E + 03
Sway (m)	0.32006E-07	0.17342E-06	0.61843E-06	-0.42536E-06
Heave (m)	-0.56944E + 01	0.11908E + 01	-0.13261E + 01	-0.10015E + 02
Roll (deg)	0.20025E-09	0.88863E-08	0.58196E-07	-0.61182E-07
Pitch (deg)	0.55386E-01	0.12915E + 01	0.45927E + 01	-0.46059E + 01
Yaw (edg)	-0.38013E-07	0.17102E-06	0.44091E-06	-0.47798E-06

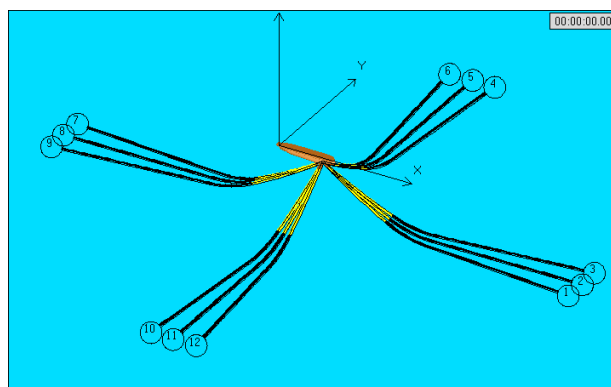


Figure 2. The layout configuration of turret mooring system

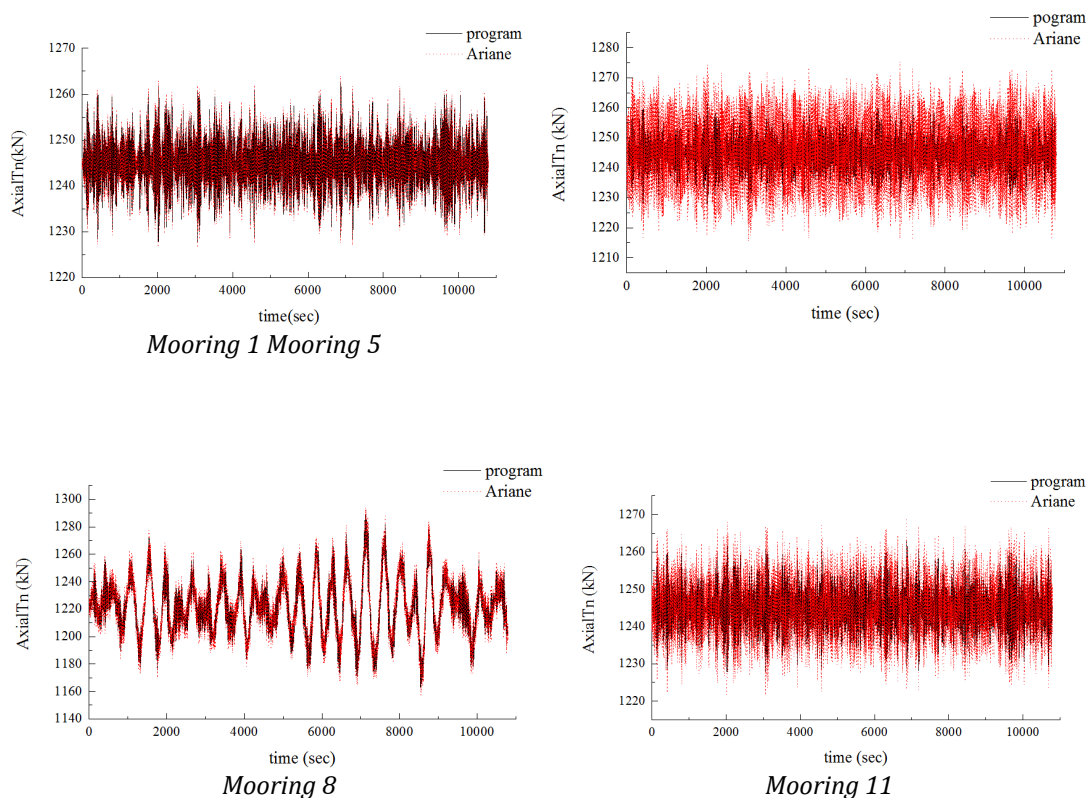


Figure 3. The mooring force comparison between our program and Ariane software

Form the table 2 it can be observed that the FPSO undergoes significant slow drift surge motion. Our program estimates the varying drift forces and the damping of hull of hydrodynamic. Form table 2 we can see the amplitude of sway and roll are small, because of the length of FPSO is so long. Form the statistic, it can be seen the angle of yaw is small because of the FPSO has the vane effect with the turret's exist. On account of the vane effect, FPSO can work in head-sea at almost any time, it provide the stability and safety of FPSO.

The hull is moored with 12 mooring lines. The mooring lines are divided into four groups with 90 degrees interval, each group consists of 3 lines with 5 degrees interval to each other. Fig. 2 shows the lay out configuration of the whole mooring system. Tab.2 presents the parameters of mooring lines.

Firstly, we obtained the hydrodynamic coefficients of FPSO by Hydro-STAR, and computed the 6DOF motions' RAO (Response Amplitude Operator). Moreover, we loaded the data from Hydro-STAR such as added mass, damping coefficient and 6DOF motions' RAO into Ariane software to compute the load of FPSO's mooring line at fairlead. Fig. 3 shows the curve of time history that is comparison of our program and Ariane software.

Firstly, the length of single mooring line is 1500 m, the mooring lines are attached at the location of turret. Fig.1 gives us the layout configuration of the mooring system. Using the dynamic response model of the FPSO, the FPSO's response in head irregular waves is simulated and compared with the software

Ariane by BV. In our program, the JONSWAP sea spectrum is used, the significant wave height  $h_s = 4.2\text{m}$ , spectral peak period  $T_p = 14.0\text{ s}$  and the spectral peak factor  $\gamma = 2.3$ . The program simulation are 54000 steps and the time step solution of FPSO is 0.2 s. The simulation is performed at heading seas.

During the comparison the irregular wave time history is generated in our program and Ariane software in order to ensure the two methods have the same incident wave time history.

The following Fig. 3 show us the comparison results of mooring system from that of our program with that of Ariane. It can be seen from the comparison that the results from our program agree quite well with that from Ariane software. The mooring tensions of 1# which is in the windward side and is the most loaded lines, the mooring tensions of 8# which is in the leeward side.

From the comparison of 1#, 5#, 8#, 11# mooring lines it is shown that our program can estimates the mooring lines force at fairleads of the FPSO are correctly. From the results of Fig. 3, it can be seen that the most loaded line shows a combination tension response from wave frequency and low frequency range.

• **The hydrodynamic analysis of FPSO in South China Sea**

In addition, our program is used to compute the hydrodynamic of FPSO in South China Sea. The environment of South China Sea simulated by the JONSWAP sea spectrum, the significant wave height

hs = 12.9 m, spectral peak period  $T_p = 16.1$  s and the spectral peak factor  $\gamma = 2.0$ . The API wind spectrum, viscous flow load by empirical equation of Ocimf and wave drift damping of floating structure are used in this example. Table 3 provide the statistic of FPSO's 6DOF motions.

From table3 it is shown the sway/roll of FPSO is quite well, and the yaw is pretty good. The mean value of yaw is  $-3.66 \times 10^{-8}$  deg, and the stander deviation is  $1.33 \times 10^{-7}$ , while the surge of FPSO is a little large, the mean value is  $-1.01 \times 10^2$  m. Because of the vane effect, the performance of yaw motion quite well, and the model length let the FPSO has a good commend of sway and roll motion. From the surge statistic shows us the FPSO in the head sea at most time.

It is needed to ensure the equilibrium position and initial form at static load when dynamic analysis of mooring system. To solve the initial position and form, then compute the equilibrium position by it.

Table 4-7 provide the load at fairlead of statistic of mooring lines by static analysis and dynamic analysis, they are mooring 2, mooring 5, mooring 8 and mooring 11. The  $F_X$   $F_Y$  and  $F_Z$  are force in x y and z directions, respectively,  $M_X$   $M_Y$  and  $M_Z$  are moment in x y and z directions, respectively, and the Axial Tn indicate the tension of axial direction of mooring line at fairlead in table 4-7.

The mooring 2 and 8 are in head-sea and following-sea, respectively, and mooring 5 and 11 are located on either side of FPSO.

Table 3. Statistic of FPSO's 6DOF motions

Motion	Mean	Std	Max	Min
Surge (m)	-1.01E + 02	9.25E + 00	-7.24E + 01	-1.30E + 02
Sway (m)	2.82E-08	1.41E-07	6.05E-07	-3.32E-07
Heave (m)	-5.69E + 00	1.18E + 00	-9.56E-01	-1.03E + 01
Roll (deg)	2.19E-10	7.35E-09	6.17E-08	-4.69E-08
Pitch (deg)	5.61E-02	1.29E + 00	5.03E + 00	-4.53E + 00
Yaw (deg)	-3.66E-08	1.33E-07	3.00E-07	-4.15E-07

Table 4. The statistic of mooring 2

Static analysis of mooring 2

	$F_X$ (N)	$F_Y$ (N)	$F_Z$ (N)	$M_X$ (N·m)	$M_Y$ (N·m)	$M_Z$ (N·m)	AxialTn (N)
Mean	923861.4	4.5E-06	-1058123	0.007299	77604534	0.042495	1405096
Std	98610.4	0.000131	60939.28	0.03279	4511886	0.190903	110853.1
Max	1292124	0.000374	-888148	0.102888	97731850	0.599015	1819079
Min	666522.4	-0.00037	-1280669	-0.09132	63758422	-0.53164	1110593

Dynamic analysis of mooring 2

	$F_X$ (N)	$F_Y$ (N)	$F_Z$ (N)	$M_X$ (N·m)	$M_Y$ (N·m)	$M_Z$ (N·m)	AxialTn (N)
Mean	812026.3	0.000859	-997083	0.012448	74013755	0.072474	1285937
Std	90642.98	0.019399	103311.5	0.287824	7573709	1.67572	137170.8
Max	1051843	0.437108	-805420	6.485114	96949361	37.75656	1651546
Min	650726.1	-0.00275	-1278645	-0.04223	60041487	-0.24585	1035445

Table 5. The statistic of mooring 5

Static analysis of mooring 5

	$F_X$ (N)	$F_Y$ (N)	$F_Z$ (N)	$M_X$ (N·m)	$M_Y$ (N·m)	$M_Z$ (N·m)	AxialTn (N)
Mean	6292.811	782581.1	-969324	11613503	83622505	67614223	1245831
Std	4135.45	15059.07	12421.77	223476.5	1387579	1301088	19124.04
Max	19413.27	838028.2	-927225	12436338	88460944	72404796	1316143
Min	-6656.44	731777.6	-1014840	10859580	78487256	63224853	1181226

Dynamic analysis of mooring 5

	$F_X$ (N)	$F_Y$ (N)	$F_Z$ (N)	$M_X$ (N·m)	$M_Y$ (N·m)	$M_Z$ (N·m)	AxialTn (N)
Mean	1736.631	779980.6	-966252	11574912	83446018	67389543	1241802
Std	7111.958	71565.83	88317.96	1062037	7721270	6183216	113645.3
Max	40820.68	974443.9	-738368	14460748	1.04E + 08	84190982	1549099
Min	-26800.2	596673	-1204549	8854627	63903322	51551947	949321.8

Table 6. The statistic of mooring 8

Static analysis of mooring 8

	F_X (N)	F_Y (N)	F_Z (N)	M_X (N·m)	M_Y (N·m)	M_Z (N·m)	AxialTn (N)
Mean	-669075	1.72E-05	-892420	-0.00718	87019072	-0.04179	1115735
Std	73428.29	0.000101	51105.04	0.022889	5266036	0.133262	84930.38
Max	-447023	0.000492	-727864	0.053329	1.05E + 08	0.310481	1441743
Min	-952639	-0.00018	-1082213	-0.10759	70947295	-0.62637	854175

Dynamic analysis of mooring 8

	F_X (N)	F_Y (N)	F_Z (N)	M_X (N·m)	M_Y (N·m)	M_Z (N·m)	AxialTn (N)
Mean	-763986	0.000474	-952906	0.007309	93649022	0.042551	1221521
Std	127094.8	0.010575	117702.9	0.157037	11690709	0.914278	172035.4
Max	-562398	0.151539	-699537	2.252462	2.92E + 08	13.11391	4272386
Min	-3177532	-0.03517	-2855971	-0.52115	69600853	-3.03418	912921.1

Table 7. The statistic of mooring 11

Static analysis of mooring 11

	F_X (N)	F_Y (N)	F_Z (N)	M_X (N·m)	M_Y (N·m)	M_Z (N·m)	AxialTn (N)
Mean	6292.811	-782581	-969324	-1.2E + 07	83622505	-6.8E + 07	1245831
Std	4135.45	15059.07	12421.77	223476.6	1387579	1301088	19124.04
Max	19413.27	-731778	-927225	-1.1E + 07	88460944	-6.3E + 07	1316143
Min	-6656.44	-838028	-1014840	-1.2E + 07	78487256	-7.2E + 07	1181226

Dynamic analysis of mooring 11

	F_X (N)	F_Y (N)	F_Z (N)	M_X (N·m)	M_Y (N·m)	M_Z (N·m)	AxialTn (N)
Mean	1736.63	-779981	-966252	-1.2E + 07	83446017	-6.7E + 07	1241802
Std	7111.957	71565.8	88317.93	1062037	7721268	6183214	113645.2
Max	40820.66	-596673	-738368	-8854627	1.04E + 08	-5.2E + 07	1549099
Min	-26800.2	-974444	-1204549	-1.4E + 07	63903322	-8.4E + 07	949321.8

It can be seen from table 4-7, the force is maximum at the fairlead when motion in the same direction with wind and wave, while the moment is the minimum, such as the force in x-axis direction in table 4 and table 6. However, the force is the minimum when motion direction and wind/wave direction angle of 90 degrees, such as force of y-axis direction in table 4 and table 6.

The force of Single Point Moored (SPM) FPSO in z-axis direction has a little change form numerical. From the numerical on the table 4 and table 6, it can be obtained that the z-axis direction's moment of mooring 2 and mooring 8 is minimum on the wave direction while the motion of mooring 5 and mooring 11 is larger than mooring 2 and mooring 8. It is not hard to see that the z-axis direction's displacement of mooring 2 and mooring 8 is negligibly small.

The statistic of four classic mooring lines is provided by table 4-7 and the force analysis at fairlead by static analysis and dynamic analysis. Based on the environment of South China Sea, the

effect of FPSO with the wave/wind/current and mooring line are considered.

Compared with the static analysis, dynamic analysis can not only consider the linear and weak nonlinear factors on the influence of the mooring system, but also to consider strongly nonlinear factors.

To estimate the loads of floating structure in the actual state of sea more accuracy, it must be used the dynamic analysis.

#### 4. Conclusion

The research develops a program to perform the dynamic coupled analysis of Single Point Moored FPSO with wind/wave/current and its mooring lines. Via comparison with the Ariane software, it can be concluded that the program in this research seems to be properly established.

Then our program is used to compute the SPM-FPSO work in South China Sea, and provide two analysis methods, they are static analysis and dynamic analysis. Furthermore, the statistic tables provide the differences of static analysis and dynamic analysis. To estimate the loads of floating structure in the actual state of sea more accuracy, it must be used the dynamic analysis.

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