SECOND ORDER DYNAMIC MODELLING OF A TRIMOBIL SCARA ROBOT USING A SYMBOLIC COMPUTATIONAL METHOD

Andreea Kurtuhuz[1], Catalin Dumitru[1]
[1]Valahia University of Targoviste, Romania
Email: catalindumitru1984@yahoo.com

Abstract - In order to command and control the activity of robots, it is necessary to know the equations of the dynamics that govern their movement. This is what we call direct dynamics. But, in addition to this, we need to know the inverse dynamics, that is, to know the law of motion of all the independent parameters, corresponding to the degrees of freedom. This paper uses an interactive software developed in the Mathematica program, which is based on a symbolic calculation method for the direct and inverse dynamics of robot manipulators that are constituted as open kinematic chains using only rotation or translation joints, with a single degree of freedom. Based on this software, this paper presents the modeling of the direct dynamics and inverse dynamics of a SCARA robot with three degrees of freedom, R-R-T.

Keywords: Robot, Manipulator, Modelling, Simulation, Computational, Symbolic.

1. Introduction

For the control of the robots it is necessary to know the equations of the dynamics, that is to say to realize the modeling of the dynamics [1]-[6].

Several different methods such as Lagrange-Euler, Newton-Euler or the generalized d'Alembert model can be used to model the dynamics of robot manipulators [7]-[9]. There are many papers on robot – manipulators modeling [10]-[15], especially kinematic [16]-[20] and dynamic modeling [21]-[23], but most deal with the case of robots with flat motion, with two or three degrees of freedom. It is recommended to use an advanced mathematical program, because the calculations are very laborious.

The Lagrange Euler method [24]-[26] is relatively simple and is suitable for symbolic calculation. when we write the dynamics equations, a series of simplifying hypotheses are made, such as neglecting the frictions in the gears, neglecting the dynamics of the control devices and we obtain systems of equations highly nonlinear with second order derivatives. These equations are deduced using the hypothesis of inertial loads, centrifugal forces and Coriolis and taking into account the effects of gravitational mass. The forces and inertia torques of each robotic arm depend on the position of its center of gravity. The forces and moments in the joints depend on the position and speed of the centers of mass of the robot arms.

The main objective of this paper is to determine the equations of the 3D trimobil SCARA robot dynamics taking into account the mechanical characteristics (masses, inertia and actuator inertia) and the operational vectors.

At the same time, the objective of the paper is to integration of the differential equation system, considering like inputs loads and actuator torques vector from robot joints.

This paper uses an algorithm made in programming language of Mathematica software which allows an advanced mathematical calculation, both symbolic and numerically, specific to modeling the dynamics of body systems.

The algorithm consists in the following steps: determining the Denavit-Hartenberg parameters for robot, design of the geometric model, design of kinematic model, design of dynamic model, dynamics simulation. It can be use for the robots with plan and spatial motion.

2. The Lagrange-Euler Dynamic Model

Using the Lagrange formalism, we obtain the dynamic Lagrange-Euler model:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_p}\right) - \frac{\partial L}{\partial q_p} = F_i \quad (i = 1,2,\ldots,n) \tag{1}$$

where

$$L = E_c - E_p \tag{2}$$

and

n – freedom degrees number of the robot;
q_i – generalized coordinates for joint i;
$\dot{q}_i$ - generalized velocity of joint i;
F_i - generalized force for joint i.

If q_i describes a linear movement, F_i will represent the necessary force to be applied to the translation joint i in order to have the desired dynamics. If q_i is the angular coordinate, F_i will be the couple of joint i.

The kinematic energy of the element i, ri being the mass position vector, is:

$$E_c = \frac{1}{2}\sum_{j=1}^{i}\sum_{k=1}^{i} \mathrm{Diag}\left[\frac{\partial\left({}^0_i[T]\right)}{\partial q_j}[J_i]\frac{\partial\left({}^0_i[T]\right)^T}{\partial q_k}\right]\dot{q}_j\dot{q}_k \quad (3)$$

where:

${}^0_i[T]$- is the passage-matrix from the system of reference attached to the i-element to the fixed-reference system;

$J_i = \int_i \vec{r}_i\, \vec{r}_i^{\,T} dm$ - is the 4x4 inertial matrix of the kinematic element i, having planar, centrifugal and static moments as components.

Diag represents the sum of the main diagonal elements.

The potential energy of the kinematic element i in the gravitational field with the mass centre determined by r_i is:

$$E_p = -\sum_{i=1}^{n} m_i [g]^T\, {}^0_i[T]^i[r_i] \quad (4)$$

m_i – the mass of the i element [kg];
g – the vector of gravitational acceleration in respect to the fixed-reference system;

Taking into account the equations (2), (3) and (4), the Lagrange function of one manipulator-robot with n kinematic elements is:

$$\begin{aligned} L &= E_c - E_p = \\ &= \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{i} \mathrm{Diag}\left[\frac{\partial\left({}^0_i[T]\right)}{\partial q_j}[J_i]\frac{\partial\left({}^0_i[T]\right)^T}{\partial q_k}\right]\dot{q}_j\dot{q}_k + \\ &+ \frac{1}{2}\sum_{i=1}^{n} I_{ai}\dot{q}_i^2 + \sum_{i=1}^{n} m_i [g]^T\, {}^0_i[T]^i[r_i] \end{aligned} \quad (5)$$

Replacing (5) in (1), results:

$$F_i = \sum_{j=1}^{n} D_{ij}\ddot{q}_j + J_{ai}\ddot{q}_i + \sum_{j=1}^{n}\sum_{k=1}^{n} D_{ijk}\dot{q}_j\dot{q}_k + D_i \quad (6)$$

where:

$$D_{ij} = \sum_{p=\max i,j}^{n} \mathrm{Diag}\left[\frac{\partial\left({}^0_p[T]\right)}{\partial q_j}[J_p]\frac{\partial\left({}^0_p[T]\right)^T}{\partial q_i}\right] \quad (7)$$

$$D_{ijk} = \sum_{p=\max i,j,k}^{n} \mathrm{Diag}\left[\frac{\partial\left({}^0_p[T]\right)}{\partial q_j \partial q_k}[J_p]\frac{\partial\left({}^0_p[T]\right)^T}{\partial q_i}\right] \quad (8)$$

$$D_i = \sum_{p=i}^{n} -m_p [g]^T \frac{\partial\left({}^0_p[T]\right)}{\partial q_i}{}^i[r_i] \quad (9)$$

with i=1, 2, ..., n.

The coefficients (7), (8), (9) have the following meaning:
D_{ii} = effective inertia of joint i
D_{ij} = D_{ji} – coupling inertia between joints i and j
D_{ijk} = D_{ikj} – Coriolis force of joint i due to speeds of joint j and k
D_i – gravitational loads of joint i
J_{ai} – actuator i inertia
J_i – pseudoinertia matrix

This inertial parameters influence the stability and the positional precision of the endeffector.

3. Algorithm - Software for Dynamic Modeling and Dynamic Simulation

The algorithm - software for modelling, based of the Denavit – Hartenberg method, has 2 parts:
1 - the algorithm - software for the *dynamic model*
2 - the algorithm - software for the *dynamic simulation*

The input data of the software for the *dynamic model*: the Denavit-Hartenberg geometric parameters, the coordinates of the mass centre for each kinematic element, the axial and centrifugal inertia of each kinematic element, the inertia of the actuators for each joint, the vector of gravitational acceleration.

The output data of the software for the dynamic model are: the dynamics equations.

The input data of the software for the *dynamic simulation*: the generalized forces for each degree of freedom, Fi; the initial conditions for positions qi(0); time interval for the interpolation.

The output data of the software for the *dynamic simulation* are: the diagrams which describe the time-evolution of the position coordinates for the robot in given time interval;
There are also two optional outputs: the evolution of the coordination of the robot for some interesting points; the visualization of the evolution for the coordinates qi(t) for an other time interval.

The notations used in the software are:
m[i] – the mass of the i element [kg];
q[i] – generalized coordinates for joint i;
g – the vector of gravitational acceleration in respect to the fixed-reference system;
F[i][t] – the generalized force of joint i.

4. Application for a 3D Trimobil SCARA Robot

Using the software described in the previous point, we will generate the equations of dynamics (direct dynamics) and we will determine the motion relations specific to each degree of freedom (inverse dynamics) for the SCARA robot with 3 degrees of freedom R-R-T (Fig.1).

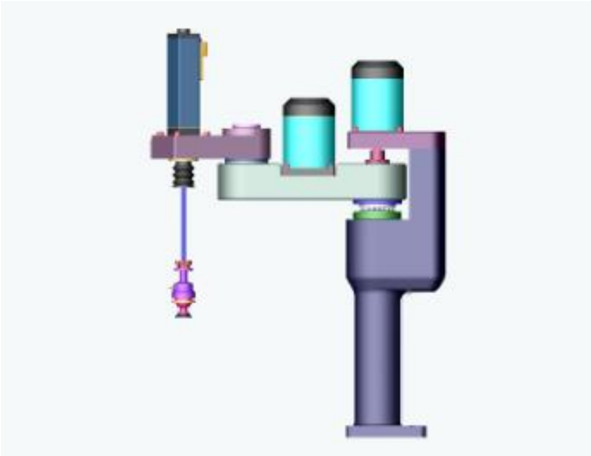

Fig. 1. SCARA Robot

This process involves several stages:
- realization of the structural scheme of the robot (Fig.2)
- writing the Denavit-Hartenberg parameters (Table1)
- calculation of the mass, mass moments of inertia and coordinates of the center of mass for each robotic element (Table2)
- running the software for dynamic modelling (direct dynamics)
- running the software for dynamic symulation (inverse dynamics)

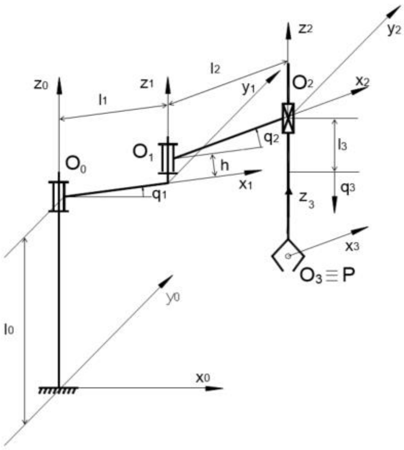

Fig. 2. Kinematic structure for the SCARA Robot

– *Writing the Denavit-Hartenberg parameters:*

Table 1. Denavit-Hartenberg parameters

	θ_i	d_i	a_i	α_i
(O_0O_1) i=1	q_1	l_0	l_1	0
(O_1O_2) i=2	q_2	h	l_2	0
(O_2P) i=3	0	$-l_3-q_3$	0	0

– *Calculation the values of the mass, the coordinates of the mass centre and the inertial matrix for each link*

Table 2. Other parameters

	Mass	x_c	y_c	z_c	I_{xx}	I_{yy}	I_{zz}
	[kg]	[m]	[m]	[m]	[Kg m^2]	[Kg m^2]	[Kg m^2]
1	137.63	0.26	0	0	1.49	17.40	26.72
2	115.36	-0.31	0	0	1.19	13.39	13.81
3	6.89	0	0	0.65	3.88	3.88	0.002

– *Running the algorithm - software for dynamic model:*

It obtains the following results:

The position vector of the characteristic point P of robot, with respect to the system $\{0\}$:

$$ {}^{0}\left[r_P\right] = \begin{pmatrix} l_1 \cos q_1 + l_2 \cos\left[q_1 + q_2\right] \\ l_1 \sin q_1 + l_2 \sin\left[q_1 + q_2\right] \\ h + l_0 - l_3 - q_3 \\ 1 \end{pmatrix} $$

The velocity of the characteristic point P of robot, with respect to the system $\{0\}$:

$$ v_x = [-l_1 \sin q_1(t) - l_2 \sin(q_1(t) + q_2(t))]\dot{q}_1(t) - \\ - l_2 \sin(q_1(t) + q_2(t))\dot{q}_2(t) $$

$$ v_y = [l_1 \cos q_1(t) + l_2 \cos(q_1(t) + q_2(t))]\dot{q}_1(t) + \\ + l_2 \cos(q_1(t) + q_2(t))\dot{q}_2(t) $$

$$ v_z = -\dot{q}_3(t) $$

The acceleration of the characteristic point P of robot, with respect to the system $\{0\}$:

$$ a_x = [-l_1 \cos q_1(t) - l_2 \cos(q_1(t) + q_2(t))]\dot{q}_1^2(t) - \\ - 2l_2 \cos(q_1(t) + q_2(t))\dot{q}_1(t)\dot{q}_2(t) - \\ - l_2 \cos(q_1(t) + q_2(t))\dot{q}_2^2(t) + [-l_1 \sin q_1(t) - \\ - l_2 \sin(q_1(t) + q_2(t))]\ddot{q}_1(t) - \\ - l_2 \sin(q_1(t) + q_2(t))\ddot{q}_2(t) $$

$$ a_y = [-l_1 \sin q_1(t) - l_2 \sin(q_1(t) + q_2(t))]\dot{q}_1^2(t) - \\ - 2l_2 \sin(q_1(t) + q_2(t))\dot{q}_1(t)\dot{q}_2(t) - \\ - l_2 \sin(q_1(t) + q_2(t))\dot{q}_2^2(t) + [l_1 \cos q_1(t) + \\ + l_2 \cos(q_1(t) + q_2(t))]\ddot{q}_1(t) + \\ + l_2 \cos(q_1(t) + q_2(t))\ddot{q}_2(t) $$

$$a_z = -\ddot{q}_3(t)$$

The velocity and the acceleration of endeffector, with respect to the system {0}:

$$\omega_x = 0 \qquad \varepsilon_x = 0$$

$$\omega_y = 0 \qquad \varepsilon_y = 0$$

$$\omega_z = \dot{q}_1(t) + \dot{q}_2(t) \qquad \varepsilon_z = \ddot{q}_1(t) + \ddot{q}_2(t)$$

The dynamics equations:

$$\begin{aligned}
F_1(t) = {} & 2[m_2(-l_1 l_2 \sin q_2(t) - l_1 \sin q_2(t) x_2 - \\
& - l_1 \cos q_2(t) y_2) + m_3(-l_1 l_2 \sin q_2(t) - l_1 \sin q_2(t) x_3 - \\
& - l_1 \cos q_2(t) y_3)]\dot{q}_1(t)\dot{q}_2(t) + [m_2(-l_1 l_2 \sin q_2(t) - \\
& - l_1 \sin q_2(t) x_2 - l_1 \cos q_2(t) y_2) + m_3(-l_1 l_2 \sin q_2(t) - \\
& - l_1 \sin q_2(t) x_3 - l_1 \cos q_2(t) y_3)]\dot{q}_2^2(t) + I_{ac1}\ddot{q}_1(t) + \\
& + [I_{zz1} + I_{zz2} + I_{zz3} + m_1(l_1^2 + 2l_1 x_1) + m_2(l_1^2 + \\
& + 2l_1 l_2 \cos q_2(t) + l_2^2 + 2l_1 \cos q_2(t) x_2 + 2l_2 x_2 - \\
& - 2l_1 \sin q_2(t) y_2) + m_3(l_1^2 + 2l_1 l_2 \cos q_2(t) + l_2^2 + \\
& + 2l_1 \cos q_2(t) x_3 + 2l_2 x_3 - 2l_1 \sin q_2(t) y_3)]\ddot{q}_1(t) + \\
& + [I_{zz2} + I_{zz3} + m_2(l_1 l_2 \cos q_2(t) + l_2^2 + l_1 \cos q_2(t) x_2 \\
& + 2l_2 x_2 - l_1 \sin q_2(t) y_2) + m_3(l_1 l_2 \cos q_2(t) + l_2^2 + \\
& + l_1 \cos q_2(t) x_3 + 2l_2 x_3 - l_1 \sin q_2(t) y_3)]\ddot{q}_2(t)
\end{aligned}$$

$$\begin{aligned}
F_2(t) = {} & [m_2(l_1 l_2 \sin q_2(t) + l_1 \sin q_2(t) x_2 + l_1 \cos q_2(t) y_2) + \\
& + m_3(l_1 l_2 \sin q_2(t) + l_1 \sin q_2(t) x_3 + \\
& + l_1 \cos q_2(t) y_3)]\dot{q}_1^2(t) + [I_{zz2} + I_{zz3} + m_2(l_1 l_2 \cos q_2(t) + \\
& + l_2^2 + l_1 \cos q_2(t) x_2 + 2l_2 x_2 - l_1 \sin q_2(t) y_2) + \\
& + m_3(l_1 l_2 \cos q_2(t) + l_2^2 + l_1 \cos q_2(t) x_3 + \\
& + 2l_2 x_3 - l_1 \sin q_2(t) y_3)]\ddot{q}_1(t) + I_{ac2}\ddot{q}_2(t) + \\
& + [I_{zz2} + I_{zz3} + m_2(l_2^2 + 2l_2 x_2) + m_3(l_2^2 + 2l_2 x_3)]\ddot{q}_2(t)
\end{aligned}$$

$$F_3(t) = -g m_3 + I_{ac3}\ddot{q}_3(t) + m_3\ddot{q}_3(t)$$

– *Running the algorithm - software for dynamic simulation* in following hypothesis:

$F_1[t] = 2 \sin(\pi t)$ [N m],

$F_2[t] = \sin(t)$ [N m],

$F_3[t] = \sin(t)$ [N],

where t is the time, and the initial conditions are: $q_1(0)=0$ rad, $q_2(0)=0$ rad, $q_3(0)=0.1$m, $q'_1(0)=0$ rad/s, $q'_2(0)=0$ rad/s, $q'_3(0)=1$m/s.

It obtains the following diagrams (Fig.3) which describe the time-evolution of the position coordinates for the robot:

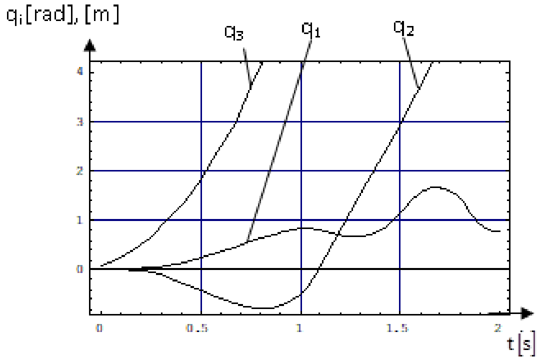

Fig. 3. Time-evolution of the position coordinates

5. Conclusions

The algorithms (routines) made in programming languages of mathematical calculation software, such as Mathematica, MathCAD, MathLAB etc. has advantages compared to classical programming languages, because they allow an advanced mathematical calculation, both in terms of symbolic calculation and in terms of numerical calculation and is very well suited to the study of the direct dynamics and inverse dynamics of multibody systems.

References

[1] Han, Yong; Wu, Jianhua; Liu, Chao,; et al., An Iterative Approach for Accurate Dynamic Model Identification of Industrial Robots, IEEE Transactions on Robotics, Volume 36 Issue , 5 2020

[2] Six D., Chriette A., https://www.sciencedirect.com/science/article/pii/S2405896317303282 - !Martinet Ph., Dynamic Modeling and Trajectory Tracking Controller of a Novel Flying Parallel Robot, IFAC-PapersOnLine, Volume 50, Issue 1, 2017

[3] Thuruthel G. et all, First-Order Dynamic Modeling and Control of Soft Robots, Frontiers in Robotics and AI, Volume 7, Article Number 95, 2020

[4] Aivaliotisa P., Papalitsaa E., G. Michalosa, S. Makrisa, Identification of dynamic robot's parameters using physics-based simulation models for improving accuracy, CIRPe 2020 – 8th CIRP Global Web Conference – Flexible Mass Customisation, 2020

[5] Paul, R. C., Robot Manipulators - Mathematics, Programming and Control, MIT Press Cambridge, 1981

[6] Fu, K., Gonzales,R., Lee,C., Control, Sensiting, Vision and Inteligence, McGraw-Hill, 1988

[7] Filip V., Zaharia S, 2001, A Symbolic Computational Method for Dynamic Simulation of Robot Manipulator, 2nd European Conference on Computational Mechanics (rang A), Krakova,

Poland, On CD-Rom, ISBN 83-85688-68-4, ECCM 2001

[8] Coiffet Ph., La Robotique: principes et application, Hermes, Paris, 1986

[9] Filip V., Dynamic modeling of manipulators with symbolic computational method, Proceedings of the Romanian Academy, Series A, Volume 9, Number 3, 2008

[10] Antonescu P., Manipulator Synthesis, University "Politehnica" of Bucharest Press, 1993

[11] Popescu P., Mechanics of Manipulators and Robots, Applications, vol. V, Dynamics, Ed. Risoprint S.R.L., Cluj - Napoca, 1997.

[12] [Filip V., Duţă L., Patic P.C., An algorithm for the direct and reverse first order kinematical analysis of the industrial robots and manipulators, Modelling and Optimization in the machines building field, MOCM–13, volume 2, Romanian Technical Sciences Academy, 2007

[13] Singh, Aditya; Pandey, Padmakar; Nandi, G. C., Effectiveness of multi-gated sequence model for the learning of kinematics and dynamics of an industrial robot, Industrial robot-the international journal of robotics research and application, Volume 48 Issue 2021 ,1

[14] Filip V., Antonescu O., Mitsi S., Geometry and direct kinematics of parallel manipulators – 6 actuator SCS type, The Scientific Bulletin of Valahia University – Materials and Mechanics, nr. 1(6), 2008

[15] Antonescu O., Filip V., Antonescu P., Topological Structure and Forward Kinematics of Parallel Manipulators with six Actuators Type RSS Fiability and Durability, nr.1, Editura "Academica Brâncuşi", Târgu Jiu, 2009

[16] Filip V., Antonescu O., Topological geometry and direct kinematics of parallel manipulators – 4 actuators, RSS type, Proceedings of the Romanian Academy, Series A, Volume 11, Number 2, 2010

[17] Filip V., Antonescu O., Mitsi S., Geometry and direct kinematics of parallel manipulators with five actuators, The Scientific Bulletin of Valahia University – Materials and Mechanics, nr. 1(6), 2008

[18] Antonescu P., Filip V., Antonescu O., Topological analysis and synthesis, geometry and kinematics of parallel manipulators, as multibody systems, Research Trends in Mechanics, ISBN 978-973-27-1816-2, p. 128-168, Editura Academiei Române, 2009

[19] Filip V., Marin A., Direct Geometry and Kinematics of Parallel Manipulators, Proceeding of International Conference „6th Workshop on European Scientific and Industrial Collaboration on promoting Advanced Technologies in Manufacturing", WESIC'08, supplement Romanian Review Precision Mechanics, Optics and Mechatronics, nr. 33, 2008

[20] Manfred l. Husty, Martin Pfurner, Hans-Peter Schrocker, A new and efficient algorithm for the inverse kinematics of a general serial 6R manipulator, Mechanism and Machine Theory 42, Elsevier, pp. 66–81, 2007

[21] Filip V., Antonescu O., Mobility and Direct Kinematics of Parallel Manipulators RSS Type – 3 Actuators, International Conference on Robotics, Cluj-Napoca, Solid State Phenomena, Vols. 166-167, p.197-202, 2010

[22] Zhang, Haodong; Zhang, Xianmin; Zhan, Zhenhui; et al., Dynamic modeling and comparative analysis of a 3-PRR parallel robot with multiple lubricated joints, International journal of mechanics and materials in design, Volume 16, issue 3, 2020

[23] Filip V., Neagu A., A Symbolic Computational Method for Dynamic Simulation of Multibody Systems, First International Conference on Innovative Computing, Information and Control – Volume I (ICICIC'06), Beijing, China, august 2006,

[24] Heydari F. et all, Analytical dynamic modeling of Delta robot with experimental verification, Proceedings of the institution of mechanical engineers part k-journal of multi-body dynamics, Volume 234, Issue 3, 2020

[25] Zaharia S., Filip V., A Symbolic Computational Method for a Dynamic Model of Robot Manipulators, 5th International Conference on Computational Structures Technology / 2nd International Conference on Engineering Computational Technology (rang B) Leuven, Belgium, sep. 06-08, 2000

[26] Urrea, C.; Pascal, J., Design and validation of a dynamic parameter identification model for industrial manipulator robots, Archive of Applied Mechanics, 2021