DETERMINATION OF DYNAMIC FORCES IN THE METAL STRUCTURE OF A TOWER CRANE BASED ON THE MULTI-MASS MODEL

Valentyn Kovalenko[1], Oleg Kovalenko[1], Vsevolod Stryzhak[1], Mariana Stryzhak[1], Andrew Ruzmetov[1]
[1]National Technical University "Kharkiv Polytechnic Institute", Ukraine
Email: vsevolod.stryzhak@khpi.edu.ua

Abstract - The article firstly reviews the current level of research on the dynamics of tower cranes. The publications covering research on the dynamics of crane systems and issues related to sway and trajectory of a load on a flexible suspension are considered. Taking into account the actual characteristics of the drive in dynamic models of tower cranes remains an urgent issue and is the aim of this study To achieve this goal, a multi-mass dynamic model of a tower crane with a slewing tower was obtained, where the drive of the slewing mechanism is taken into account as a separate mass. This approach is the main scientific novelty of this article and enables determination of the forces in elastic connections and other dynamic and kinematic characteristics of the movement depending on changes in the control parameter of the slewing mechanism motor. At the same time, it is possible to take into account the change in torque through the equation of mechanical characteristics, which allows to additionally take into account the influence of internal processes in the drive. When writing the system of equations describing this system, the Lagrange equations of the second kind were used. The resulting mathematical model takes into account the inertial (moments of inertia, reduced masses), elastic and damping characteristics of the structure. The complex movement of the load in both planes is considered. A numerical calculation is performed based on the parameters of a real crane. The change in the drive control parameter during the working cycle is modelled according to different laws.

Keywords: Tower crane, Slewing mechanism, Dynamics.

1. Introduction

Starting and braking in the mechanisms of load-handling machines can be carried out by means of frequency control of the motor current. By selecting the law of changing the current frequency, we can improve the operation of the crane mechanism and reduce the load on its individual elements. For this pupose, an adequate mathematical model should be developed that would take into account the motor, the main masses, inertia, and damping characteristics of the crane. This article considers the creation of a multi-mass crane model and its design in MATLAB.

The aim of the study is to determine the regularities of change in dynamic forces in the elements of the metal structure of a tower crane with a slewing tower for various laws of change in the control parameter of the slewing mechanism.

The research methodology is based on the principles of theoretical mechanics and numerical modelling. To construct equations describing the motion of a mechanical system, the Lagrange method is used.

2. Literature Review

The dynamics of crane mechanical systems is devoted to a large number of scientific works. This indicates the great relevance of the problem under consideration. However, it also requires a thorough literature review to determine the previously unsolved problem within the research topic.

Literature review on the research topic shows that several directions can be distinguished in this issue. This is due to the different tasks and approaches for which different crane models are synthesised.

The simplest models take into account the vibrations of the load. The remaining masses are generalised and represented as the mass of the suspension current. In this case, the mass of the suspension point can have either linear or rotational motion. Due to the simplicity of the models, they can replace each other and their purpose is primarily to build controllers for damping the load oscillations.

The article [1] presents a non-linear model of a crane trolley. The equations are composed in independent generalized coordinates: the movement of the cart and the angle of deviation of the load.

The controlling parameter is the driving force, which is applied through the rope of the traction winch.

In the article [2], the problems of the moving load are considered within the framework of the structural dynamic analysis of a large gantry crane as a high-performance machine. To obtain a mathematical model of the crane, emphasis is placed on the combined method - the method of finite elements and analytical expressions. A moving cart is considered in several models: a moving force, a moving mass, a moving oscillator, and a moving oscillator with an oscillating object. Each model has characteristics that determine the response of the crane structure, as well as its dynamic properties. The problem was solved by calculating the reactions to forced vibration of a two-dimensional structure with time-dependent property matrices and an equivalent moving load.

In work [3] a mathematical model of a pendulum with a variable suspension length of the load was obtained on the basis of LaGrange equations in order to simulate simultaneous lifting and horizontal movement of the load.

Mathematical models of such systems are also studied and can be found in [4], where modeling of the dynamics of the "trolley-load on a flexible suspension" system was carried out with different approaches to the problem statement.

The disadvantage of such studies is that only the interaction of the trolley and load is considered, without taking into account dynamic phenomena in the elements of drive mechanisms and the metal structure of the load-bearing elements of the crane.

In work [5] a mathematical model of the trolley-load system for a cable crane is obtained, which allows taking into account the curvature of the load-bearing rope, as well as the forces of resistance to movement in the presence of friction and wind.

If the system cannot be represented in trolley-load form because design features introduce too much error, it becomes more complex. The following are a number of works that take such features into account.

The model of transporting long cargo by two bridge cranes located at different levels in height and working together [6] deserves attention.

If the mass of the suspended load is strongly distributed over the height, the model of the trolley with a double pendulum is considered [7].

A lot of works are devoted directly to the consideration of the dynamics of tower cranes.

In the article [8], the dynamic and kinematic loads of a jib crane when lifting a load are investigated. Lagrange's equation of the second kind was used to construct the mathematical model. However, dissipative forces are also not taken into account in this study of dynamic loads.

The article [9] presents the modeling of a crane with a flexible rotary boom, which is actuated by hydraulic cylinders and is modeled as a flat system. The dynamic relationships of the masses of the crane and hydraulic cylinders are presented using the graph method. In addition, the procedure for determining reaction forces in passive connections is presented. Both presented methods are applied to a group of flat manipulators. The simulation results are confirmed by finite element analysis in the ANSYS simulation environment. The model is the basis for the design of the slewing mechanism cylinders and can potentially be used to study the performance of the crane control system.

Based on the theory of longitudinal and transverse bending, the authors of the article [10] established the differential equations of lift and deflection of an n-stage boom with supporting ropes. The Levenberg-Marquardt optimization algorithm is used to solve the critical force and length factor. The analysis of the errors of the results confirmed the correctness of the derived equation of the bending characteristic and its derivative formula, as well as the accuracy of the numerical solution. The numerical solution method can be used in the design of the structure of a telescopic boom with a lifting device in practical projects.

A detailed study based on LaGrange equations can be found in works by Perig. In [11], a model describing the oscillations of a spherical pendulum is obtained.

Models based on LaGrange equations for tower cranes are described in the works by Loveikin et al. In [12], the mechanism of sweep of the load of a tower crane is considered. The movement of the load trolley, which is moved along the horizontal saddle jib, is optimized. Optimization is performed using the Euler-Poisson equation with the integral criterion. Thus, dynamic loads and energy losses are reduced.

In [13], a dynamic model of a rotary tower crane with a saddle jib and a movable trolley with a load on a flexible suspension is considered. The equations of motion of this mechanical system, which are Lagrange equations, are constructed. Tangential and radial oscillation of the load are studied. The main attention is focused on dynamic processes associated with low-frequency oscillations caused by the load on the flexible suspension and the drive of the crane rotation mechanism. Dynamic processes in the links of the metal structure of the crane are not studied.

The article [14] investigates the dynamics of a two-dimensional structure of a jib crane, which is affected by the movement of a cargo trolley and a load fixed on a flexible element. The finite element method and the direct integration method were used to study dynamic loads. Numerical results show that the used approach is useful and can be used to modernize existing and design new jib cranes.

The main aims of crane dynamics researches are to determine the maximum stresses in the steel structure and consequently the service life. These questions also attract the attention of researchers quite often.

The issue of studying the residual life and damage of a crane that is in operation is considered in work [15]. The issue of studying effective stresses is also considered in work [16]. However, with this approach, only the consequences of load perception can be assessed. To reduce the amount of fatigue damage, it is necessary to study dynamic processes, since the acting stresses determine the rate of damage accumulation.

In the article [17], a mathematical model is considered, which describes the multi-axial movement of the machine during movement over bumps. The generalized coordinates are: vertical and angular displacement of the center of gravity in the lateral plane. In the process of modeling, it was found that sufficient damping of oscillations is achieved much earlier than with extreme aperiodic motion.

The article [18] provides a description of the experimental design concept of the test bench, which allows conducting a series of studies of dynamic effects associated with lifting a load. Built on the basis of the proposed design of the test bench, it allows to measure such data as the acceleration of the center of the beam and the load, the displacement of the load and the center of the beam, etc. The electrical and electronic part of the test bench is made using a series of PHIDGET circuits, which are complete systems based on the USB interface, based on almost any programming language. This allows basically full control of the system by means of a PC and allows immediate data analysis, which is very important in the verification of the phenomenological model of the proposed lifting mechanism.

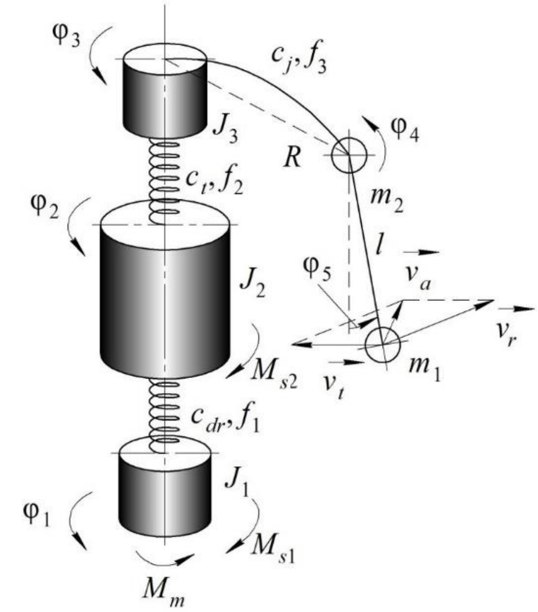
The work [19] presents the development of a controller that reduces the swaying of cargo on a floating crane. To study the oscillations, a multi-mass dynamic model of a crane was used, consisting of a rotary column and a two-link boom with a load on a flexible suspension. The swing of the load occurs in two planes. The model also takes into account the masses of hydraulic cylinders.

Thus, existing mathematical models focus on dynamic processes associated with low-frequency load oscillations. The influence of drives, their control laws and dynamic processes in elastic links are not still sufficiently studied.

3. Research Methodology

3.1. Mathematical Model

Fig. 1 shows the five-mass model of the tower crane that we use for calculations.



φ_1 - φ_5 – rotation angles (degrees of freedom);
m_1, m_2 – load weight and reduced boom weight;
J_1 - J_3 – moments of inertia of crane parts;
c_{dr}, c_t, c_j – drive, tower and boom stiffness;
f_1, f_2, f_3 – damping coefficients of the drive, tower and boom;
R – boom reach;
l – load suspension length;
v_a, v_r, v_t – absolute, relative and portable load speeds;
M_m – engine torque;
M_{s1}, M_{s2} – moments of resistance

Figure 1: Five-mass crane model

Let us start developing a mathematical model by writing Lagrange equations of the second kind. The kinetic energy for the first four degrees of freedom is written as:

$$T_K = \frac{J_K \cdot \dot{\varphi}_K^2}{2} \left(K = 1,\ 4\right) \tag{1}$$

where $J_4 = m_2 \cdot R^2$. For the fifth degree of freedom, it has the form:

$$T_5 = \frac{m_1 \cdot V_a^2}{2}. \tag{2}$$

Relative and portable speeds are related to angular velocities with relationships: $v_t = \dot{\varphi}_3 \cdot R$; $v_r = \dot{\varphi}_5 \cdot l$. Hence, the expression for the square of the absolute velocity will be as follows:

$$V_a^2 = V_t^2 + V_r^2 + 2 \cdot V_t \cdot V_r \cdot \cos(180° - \varphi_5) = \dot{\varphi}_3^2 \cdot R^2 + \dot{\varphi}_5^2 \cdot l^2 + 2 \cdot R \cdot l \cdot \dot{\varphi}_3 \cdot \dot{\varphi}_5 \cdot \cos(180° - \varphi_5) = \dot{\varphi}_3^2 \cdot R^2 + \dot{\varphi}_5^2 \cdot l^2 - 2 \cdot R \cdot l \cdot \dot{\varphi}_3 \cdot \dot{\varphi}_5 \cdot \cos(\varphi_5) \quad (3)$$

Then the expression for kinetic energy for the fifth degree of freedom will have the form:

$$T_5 = \frac{m_1 \cdot V_a^2}{2} = \frac{m_1 \cdot \left[\dot{\varphi}_3^2 \cdot R^2 + \dot{\varphi}_5^2 \cdot l^2 - 2 \cdot R \cdot l \cdot \dot{\varphi}_3 \cdot \dot{\varphi}_5 \cdot \cos(\varphi_5) \right]}{2} \quad (4)$$

and the total kinetic energy of the entire system will be:

$$T_5 = \frac{1}{2} \cdot \left(J_1 \cdot \dot{\varphi}_1^2 + J_2 \cdot \dot{\varphi}_2^2 + J_3 \cdot \dot{\varphi}_3^2 + J_4 \cdot \dot{\varphi}_4^2 + m_1 \cdot \left[\dot{\varphi}_3^2 \cdot R^2 + \dot{\varphi}_5^2 \cdot l^2 - 2 \cdot R \cdot l \cdot \dot{\varphi}_3 \cdot \dot{\varphi}_5 \cdot \cos(\varphi_5) \right] \right) \quad (5)$$

Here $g = 9{,}81 \text{ m/s}^2$. Potential energy of the system:

$$V = -m_1 \cdot g \cdot l \cdot \cos(\varphi_5) + \frac{c_{dr}}{2} \cdot (\varphi_2 - \varphi_1)^2 + \frac{c_t}{2} \cdot (\varphi_3 - \varphi_2)^2 + \frac{c_j}{2} \cdot (\varphi_4 - \varphi_3)^2 \quad (6)$$

Rayleigh function of energy dissipation:

$$F = \frac{f_1}{2} \cdot (\dot{\varphi}_2 - \dot{\varphi}_1)^2 + \frac{f_2}{2} \cdot (\dot{\varphi}_3 - \dot{\varphi}_2)^2 + \frac{f_3}{2} \cdot (\dot{\varphi}_4 - \dot{\varphi}_3)^2 \quad (7)$$

For our mechanical system, Lagrange equations of the second kind have the form:

$$\frac{d}{dt} \cdot \left(\frac{\partial T}{\partial \dot{\varphi}_K} \right) - \frac{\partial T}{\partial \varphi_K} = Q_{\varphi_K} \quad (K = 1,\ 5) \quad (8)$$

where generalized nodal forces have a structure:

$$\begin{cases} Q_{\varphi 1} = -\frac{\partial V}{\partial \varphi_1} - \frac{\partial F}{\partial \dot{\varphi}_1} + M_m - M_{s1} \\ Q_{\varphi 2} = -\frac{\partial V}{\partial \varphi_2} - \frac{\partial F}{\partial \dot{\varphi}_2} - M_{s2} \\ Q_{\varphi 3} = -\frac{\partial V}{\partial \varphi_3} - \frac{\partial F}{\partial \dot{\varphi}_3} \\ Q_{\varphi 4} = -\frac{\partial V}{\partial \varphi_4} - \frac{\partial F}{\partial \dot{\varphi}_4} \\ Q_{\varphi 5} = -\frac{\partial V}{\partial \varphi_5} \end{cases} \quad (9)$$

and are written as:

$$\begin{cases} Q_{\varphi 1} = c_{dr} \cdot (\varphi_2 - \varphi_1) - f_1 \cdot (\dot{\varphi}_2 - \dot{\varphi}_1) + M_m - M_{s1} \\ Q_{\varphi 2} = -c_{dr} \cdot (\varphi_2 - \varphi_1) + c_t \cdot (\varphi_3 - \varphi_2) - f_1 \cdot (\dot{\varphi}_2 - \dot{\varphi}_1) + f_2 \cdot (\dot{\varphi}_3 - \dot{\varphi}_2) - M_{s2} \\ Q_{\varphi 3} = -c_t \cdot (\varphi_3 - \varphi_2) + c_j \cdot (\varphi_4 - \varphi_3) - f_2 \cdot (\dot{\varphi}_3 - \dot{\varphi}_2) + f_3 \cdot (\dot{\varphi}_4 - \dot{\varphi}_3) \\ Q_{\varphi 4} = -c_j \cdot (\varphi_4 - \varphi_3) - f_3 \cdot (\dot{\varphi}_4 - \dot{\varphi}_3) \\ Q_{\varphi 5} = -m_1 \cdot g \cdot l \cdot \sin(\varphi_5) \end{cases} \quad (10)$$

After calculating partial derivatives of T for $\dot{\varphi}_k$ and φ_k, and then the complete time derivatives, taking into account (10), equations (8) take the form as follows:

$$\begin{cases} J_1 \cdot \ddot{\varphi}_1 = f_1 \cdot (\dot{\varphi}_2 - \dot{\varphi}_1) + c_{dr} \cdot (\varphi_2 - \varphi_1) + M_m - M_{s1} \\ J_2 \cdot \ddot{\varphi}_2 = -f_1 \cdot (\dot{\varphi}_2 - \dot{\varphi}_1) + f_2 \cdot (\dot{\varphi}_3 - \dot{\varphi}_2) - c_{dr} \cdot (\varphi_2 - \varphi_1) + c_t \cdot (\varphi_3 - \varphi_2) - M_{s2} \\ J_3^* \cdot \ddot{\varphi}_3 + m_1 \cdot \left[R \cdot l \cdot \sin(\varphi_5) \dot{\varphi}_5^2 - R \cdot l \cdot \cos(\varphi_5) \ddot{\varphi}_5 \right] = -f_2 \cdot (\dot{\varphi}_3 - \dot{\varphi}_2) + f_3 \cdot (\dot{\varphi}_4 - \dot{\varphi}_3) - c_t \cdot (\varphi_3 - \varphi_2) + c_j \cdot (\varphi_4 - \varphi_3) \\ J_4 \cdot \ddot{\varphi}_4 = -f_3 \cdot (\dot{\varphi}_4 - \dot{\varphi}_3) - c_j \cdot (\varphi_4 - \varphi_3) \\ J_5 \cdot \ddot{\varphi}_5 - m_1 \cdot R \cdot l \cdot \cos(\varphi_5) \ddot{\varphi}_3 = -m_1 \cdot g \cdot l \cdot \sin(\varphi_5) \end{cases} \quad (11)$$

where: $J_3^* = J_3 + m_1 \cdot R^2$; $J_3^* = J_3 + m_1 \cdot R^2$.

Motor torque M_m, which is included in the system of nonlinear differential equations (11), in turn, is related to the current frequency ω with the system of differential equations:

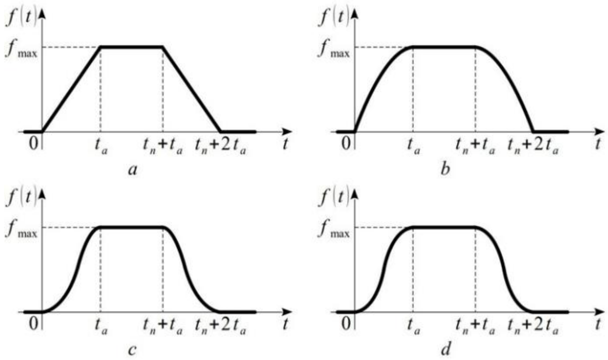
$$\begin{cases} \dot{M}_m = \frac{\beta(\omega_0 - \omega) - M_m}{T_c}; \\ \dot{\omega} = \frac{M_m - M_{st}}{J_\Sigma}, \end{cases} \quad (12)$$

where:

- $\omega_0(t) = \frac{2\pi}{p} f(t)$
- $p = 2$ - number of pairs of poes of the electric motor;
- $f(t)$ - a given law of changing the current frequency during the operating cycle of the mechanism;

- T_c - electric constant;
- β - coefficient of rigidity of linearized mechanical characteristic;
- M_{st} - moment of static resistance of rotation reduced to the motor shaft;
- J_{Σ} - total moment of inertia of the mechanism reduced to the motor shaft.

The task is to solve a system of nonlinear differential equations (11-12) under zero initial conditions and under various laws of changing the current frequency during the operating cycle $f(t)$.



a – linear; b – quadratic; c –cubic;
d – trigonometric

Figure 2: Changes in the current frequency during the operating cycle

The following laws of changing the current frequency during the operating cycle were studied during the working cycle (Fig. 2):

- linear (Fig. 2.a):

$$f(t)=\begin{cases} 0; \quad t \notin \left[0; t_n+2t_a\right]; \\ \dfrac{f_{\max}}{t_a}t; \quad t\in\left[0; t_a\right]; \\ f_{\max}; \quad t\in(t_a; t_n+t_a); \\ \dfrac{f_{\max}}{t_a}(t_n+2t_a-t); \\ \qquad t\in\left[t_n+t_a; t_n+2t_a\right]. \end{cases} \tag{13}$$

- quadratic (Fig. 2.b):

$$f(t)=\begin{cases} 0; \quad t \notin \left[0; t_n+2t_a\right]; \\ f_{\max}-\dfrac{f_{\max}(t-t_a)}{t_a^2}; \quad t\in\left[0; t_a\right]; \\ f_{\max}; \quad t\in(t_a; t_n+t_a); \\ f_{\max}-\dfrac{f_{\max}(t_n+t_a-t)^2}{t_a^2}; \\ \qquad t\in\left[t_n+t_a; t_n+2t_a\right]. \end{cases} \tag{14}$$

- cubic (Fig. 2.c):

$$f(t)=\begin{cases} 0; \quad t \notin \left[0; t_n+2t_a\right]; \\ \dfrac{f_{\max}t^2}{t_a^3}(3t_a-2t); \quad t\in\left[0; t_a\right]; \\ f_{\max}; \quad t\in(t_a; t_n+t_a); \\ \dfrac{f_{\max}(t_n+2t_a-t)^2(2t-t_a-2t_n)}{t_a^3}; \\ \qquad t\in\left[t_n+t_a; t_n+2t_a\right]. \end{cases} \tag{15}$$

- trigonometric (Fig. 2.d):

$$f(t)=\begin{cases} 0; \quad t \notin \left[0; t_n+2t_a\right]; \\ f_{\max}(\dfrac{t}{t_a}-\dfrac{1}{2\pi}\sin\dfrac{2\pi t}{t_a}); \quad t\in\left[0; t_a\right]; \\ f_{\max}; \quad t\in(t_a; t_n+t_a); \\ f_{\max}(\dfrac{t_n+2t_a-t}{t_a}-\dfrac{1}{2\pi}\sin\dfrac{2\pi(t_n-t)}{t_a}); \\ \qquad t\in\left[t_n+t_a; t_n+2t_a\right]. \end{cases} \tag{16}$$

Here $f(t)$ is rated current frequency; t_a is same acceleration and deceleration times; t_n is operating time in rated mode. All these values are given.

3.2. Input Data and Calculation Scheme

Input data for calculations were taken from a real crane:

- $R=25$ m;
- $l=25$ m;
- $m_1=4490$ kg;
- $m_2=480$ kg;
- $J_1=2,6\cdot10^5$ kg·m²;
- $J_2=5,6\cdot10^5$ kg·m²;
- $J_3=3,6\cdot10^5$ kg·m²;
- $c_{dr}=3,48\cdot10^7$ N·m;
- $c_t=4,6\cdot10^6$ N·m;
- $c_j=1,95\cdot10^5$ N·m;
- $f_1=1,07\cdot10^6$ N·m·s;
- $f_2=1,2\cdot10^6$ N·m·s;
- $f_3=1,1\cdot10^5$ N·m·s;
- $M_{s1}=4,28\cdot10^4$ N·m;
- $M_{s2}=7200$ N·m;
- $M_{st}=1,562$ N·m;
- $T_c=0,01$ s;
- $\beta=7,065$ kg·m²/s;

- $J_\Sigma = 1{,}193\ \text{kg}\cdot\text{m}^2$;
- $f_{\max} = 50\ \text{Hz}$;
- $t_a = 3{,}17\ \text{s}$;
- $t_n = 3{,}0\ \text{s}$.

The calculations were performed in the MATLAB mathematical Package. The system of differential equations (11-12) was reduced to the rated form by introducing new variables:

$$\begin{cases} q_1 = \varphi_1; \quad q_2 = \varphi_2; \quad q_3 = \varphi_3; \quad q_4 = \varphi_4; \\ q_5 = \varphi_5; \quad q_6 = \dot{\varphi}_1; \quad q_7 = \dot{\varphi}_2; \quad q_8 = \dot{\varphi}_3; \\ q_9 = \dot{\varphi}_4; \quad q_{10} = \dot{\varphi}_5; \quad q_{11} = M_m; \quad q_{12} = \omega. \end{cases} \tag{17}$$

With these notations, the system of differential equations (11-12) is written as a system of 12 normal equations:

$$\begin{cases} \dot{q}_1 = q_6 \\ \dot{q}_2 = q_7 \\ \dot{q}_3 = q_8 \\ \dot{q}_4 = q_9 \\ \dot{q}_5 = q_{10} \\ J_1\dot{q}_6 = f_1(q_7 - q_6) + c_{dr}(q_2 - q_1) + \\ \quad + q_{11} - M_{s1} \\ J_2\dot{q}_7 = -f_1(q_7 - q_6) + f_2(q_8 - q_7) - \\ \quad - c_{dr}(q_2 - q_1) + c_r(q_3 - q_2) - M_{s2} \\ J_3^*\dot{q}_8 = -(m_1 Rl\cos q_5)\dot{q}_{10} = \\ \quad = -(m_1 Rl\cos q_5)q_{10}^2 - f_2(q_8 - q_7) + \\ \quad + f_3(q_9 - q_8) - c_r(q_3 - q_2) + \\ \quad + c_j(q_4 - q_3) \\ J_4\dot{q}_9 = -f_3(q_9 - q_8) - c_j(q_4 - q_3) \\ J_5\dot{q}_{10} = -(m_1 Rl\cos q_5)\dot{q}_8 = -m_1 gl\sin q_5 \\ \dot{q}_{11} = \dfrac{1}{T_c}\left(\beta\left(\dfrac{2\pi}{p}f(t) - q_{12}\right) - q_{11}\right) \\ \dot{q}_{12} = \dfrac{1}{J_\Sigma}(q_{11} - M_{st}) \end{cases} \tag{18}$$

This system can be written in matrix-vector form as follows:

$$M(t,q)\dot{q} = f(t,q); \tag{19}$$

where q is vector-column of variables; $M(t,q)$ is a mass matrix; its nonzero components are as follows:

$$\begin{aligned} & M_{1.1} = M_{2.2} = M_{3.3} = M_{4.4} = M_{5.5} = 1; \\ & M_{6.6} = 1; \quad M_{7.7} = J_2; \\ & M_{8.8} = J_3^* \quad M_{8.10} = -m_1 Rl\cos q_5; \\ & M_{9.9} = J_4; \\ & M_{10.8} = m_1 Rl\cos q_5;\ M_{10.10} = J_5; \\ & M_{11.11} = M_{12.12} = 1. \end{aligned} \tag{20}$$

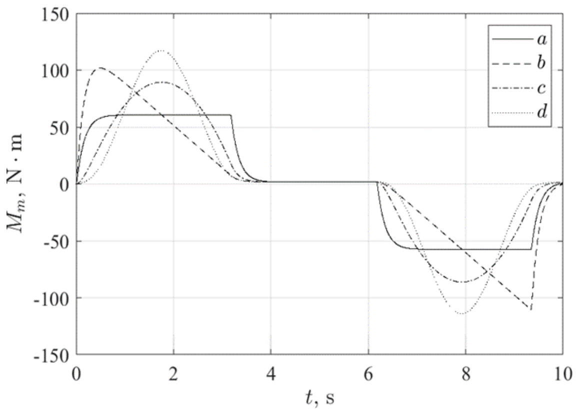
$f(t,q)$ is a column vector of the right parts of the system (18). The initial conditions for the system of differential equations (19) are zero:

$$q(0) = 0 \tag{21}$$

To solve the Cauchy problem (19, 21), the ode45 function was used with slightly modified options: absolute and relative accuracy were increased to 10^{-12}.

4. Results

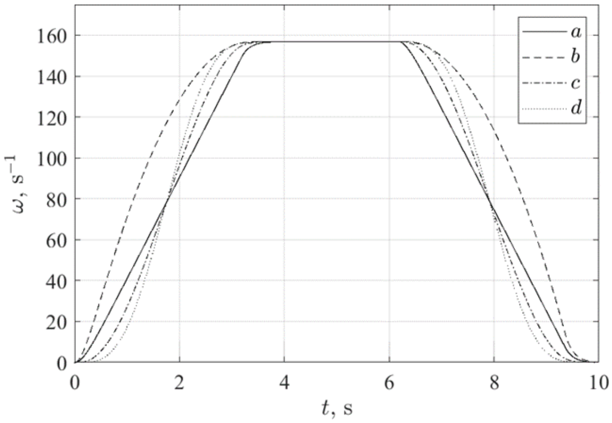
First, the dependence of motor torque on different acceleration modes was investigated. Fig. 3 shows the dependency graph $M_m(t)$ for various functions of time dependence of frequency $f(t)$. Tags a-d correspond to the acceleration modes shown in Fig. 2. We can see that the smoothest mode is the mode with the cubic law of change $f(t)$ (15).



a – linear; b – quadratic; c –cubic; d – trigonometric

Figure 3: Dependence $M_m(t)$ for different operating modes

Change graph $\omega(t)$ for different operating modes is shown in Fig. 4. These curves actually repeat the graphs from Fig. 2 taking into account the integration of the system (18).



a – linear; b – quadratic; c –cubic; d – trigonometric

Figure 4: Dependence $\omega(t)$ for different operating modes

Next, we investigate the forces in the moving elements of the crane. This depends on time t of values $c_{dr}(\varphi_2-\varphi_1)$, $c_t(\varphi_3-\varphi_2)$ and $c_j(\varphi_4-\varphi_3)$. They are shown in Fig. 5, 6, 7.

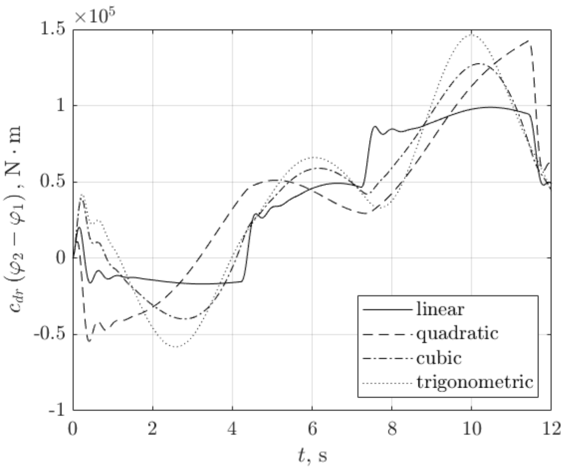


Figure 5: Time dependence $c_{dr}(\varphi_2-\varphi_1)$ for different operating modes

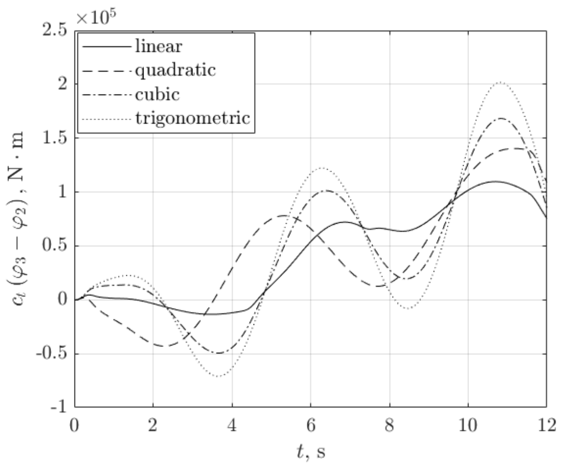


Figure 6: Time dependence $c_t(\varphi_3-\varphi_2)$ for different operating modes

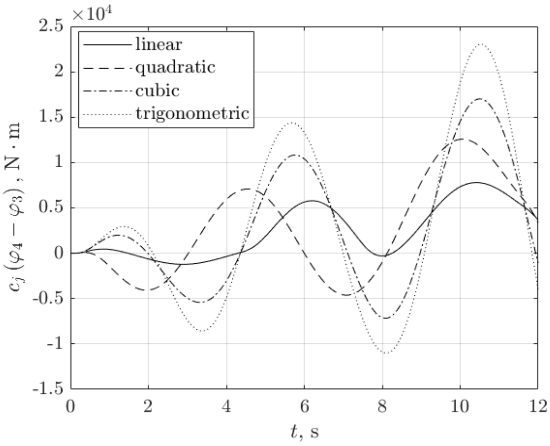


Figure 7: Time dependence $c_j(\varphi_4-\varphi_3)$ for different operating modes

5. Discussion

The article obtains a five-mass dynamic model of a tower crane with a rotating tower, which consists of Lagrange equations of the 2nd kind. The first mass represents the drive of the slewing mechanism and is affected by the motor torque. The change in the speed of the first mass depends on the control laws and changes in the control parameter. For a variable frequency drive, for example, this parameter is the frequency of the current of the motor. This approach makes it possible to more accurately determine the influence of the operating features of different drives, for example, the response of the drive to the control action, which depends on the rigidity of the mechanical characteristic. In this study, the most commonly used in practice forms of change in current frequency are modeled: linear, quadratic, S-shaped (in two versions: cubic and trigonometric).

For numerical calculations, the parameters of a real crane were taken. The different nature of the change in torque on the shaft of motor is caused by different changes in the control parameter over time. In this case, the maximum torque developed by the motor is in a large range.

The calculation showed that the maximum range of dynamic load in elastic links is determined to the greatest extent by low-frequency vibrations of the swinging load. This can also be explained by the high stiffness of the links. As the rigidity of the structure decreases, the high-frequency component will acquire greater influence and have a stronger effect on the shape of force vibrations in the elastic links.

The advantages of this study include the receipt of a dynamic model and the corresponding system of equations, which make it possible to take into account not only the inertia and rigidity of the structure, but also the mechanical characteristics of the motor of the turning mechanism. Unlike the articles mentioned in the literature review, where the dynamics of a tower crane are considered either from the point of view of oscillatory processes in the ropes or the simplified movement of the boom and load rotary system, this work examines the interaction of the elements of the "drive - rotating platform - tower - boom - load" system". In addition, the force factors acting on the load are most fully described, in contrast to some studies in which vibrations of the load are taken into account only in a plane tangent to the circular trajectory of the load when the tower rotates. The patterns of changes in forces in the elastic elements of this system were obtained, taking into account the influence of the masses of the load on the ropes and the nature of the dynamic characteristics of the starting and braking processes in the drive.

The disadvantage of this study is the high rigidity of the elements of the structure under consideration and the method of numerical integration, which do not allow us to fully assess the influence of the nature of the change in the torque of the turning mechanism motor, which can be considered an area for further research.

6. Conclusions

The mathematical model of the tower crane has been improved by increasing the number of reduced masses and taking into account the dynamical response of the drive.

Regularities of time variation of the control parameter, torque and forces in the elastic links of the metal structure are determined, their nature and magnitude are assessed. It is determined that the maximum torque can change by more than 2 times, depending on the regularity of time variation of the control parameter.

References

[1] Cakan A., Umit O. Position regulation and sway control of a nonlinear gantry crane system. International journal of scientific & technology research. 2016 Vol. 5 (11). P. 121-124.

[2] Gańić V., Zrnić Z., Milovančević M. Considerations of various moving load models in structural dynamics of large gantry cranes. FME Transactions. 2013 Vol. 41 (4). Pp. 311-316.

[3] Arabasi S., Masoud Z. Simultaneous travel and hoist maneuver input shaping control using frequency modulation. Hindawi Shock and Vibration. 2016. Vol. 10 (3). P. 179-188.

[4] Grigorov O., Druzhynin E., Anishchenko G., Strizhak M., & Strizhak V. (2018). Analysis of various approaches to modeling of dynamics of lifting-transport vehicles. International Journal of Engineering and Technology (UAE), 7(4). https://doi.org/10.14419/ijet.v7i4.3.19553

[5] Grigorov O., Okun A. Improvement of the «carriage-cargo» system motion mathematical model for solving the problem of lifting and transport machines control. Automobile transport. - 2017. - issue. 40. - pp. 120-124.

[6] Perig A. V., Stadnik A. N., Kostikov A. A. Research into 2D dynamics and control of small oscillations of a cross-beam during transportation by two overhead cranes and all. Hindawi Shock and Vibration. 2017. Vol. 12 (1). – P. 1-21.

[7] O'Connor W., Habibi H. Gantry crane control of a double-pendulum, distributed-mass load, using mechanical wave concepts. Mechanical Sciences. 2013. Vol. 4. P. 251-261.

[8] Doçi I., Hamidi B., Shpetim L. Dynamic analysis and control of jib crane in case of jib luffing motion using modelling and simulations. IFAC-PapersOnLine Vol. 49-29. 2016. Pp. 163-168.

[9] Cibicik A., Pedersen E., Egeland O. Dynamics of luffing motion of a flexible knuckle boom crane actuated by hydraulic cylinders. Mechanism and Machine Theory. Vol. 143. 2020. Pp- 1-12.

[10] Fenglin Y., Jiandong Li, Hao Y., Changkai X., Shining L. Numerical solution of critical force of n-step telescopic boom with superlift device. AIP Advances. 2023 Vol. 21. Pp 1-12.

[11] Perig A. V., Stadnik A. N., Deriglazov A. I. Spherical pendulum small oscillations for slewing crane motion. Hindawi Shock and Vibration. – 2014. – Vol. 2. – P. 24-31. doi: https://doi.org/10.1155/2014/451804.

[12] Loveikin V.S., Romasevich Y.O., Stekhno O.V. Optimization of the movement mode of the mechanism for changing the departure of the tower crane cargo with a horizontal boom. Mechanical Engineering, 2017, No. 20

[13] Loveikin V., Romasevych Y., Kurka V., Mushtyn D., Pochka K. Analysis of the start-up process of the tower crane slewing mechanism with a steady state motion mode of its load trolley. Strength of Materials and Theory of Structures – 2020. № 105

[14] Vlada Gaši, Nenad Zrni, Marko Rakin. Consideration of a moving mass effect on dynamic behaviour of a jib crane structure. Analiza utjecaja pokretne mase na dinami konstrukcije čko ponašanje stupne konzolne

dizalice. Tehni ki vjesnik. Vol. 19, 1. 2012. Pp 115-121.

[15] Gubskyi, S., Yepifanov, V., Chukhlib, V., Basova, Y., Okun, A., Ivanova, M., Panamariova, O. “Integrated Approach to Determine Operational Integrity of Crane Metal Structure”, Periodica Polytechnica Mechanical Engineering, 63(4), pp. 319–325, 2019. https://doi.org/10.3311/PPme.14064

[16] Sergienko, N., Hubskyi, S., Pavlova, N., Turchyn, O., Hasiuk, O., Židek, K. (2023). Obstacle-Resistant Wireless Strain Gauge Complex for Automated Monitoring of the Steel Structures Condition. In: Balog, M., Iakovets, A., Hrehova, S. (eds) EAI International Conference on Automation and Control in Theory and Practice . EAI ARTEP 2023. EAI/Springer Innovations in Communication and Computing. Springer, Cham. https://doi.org/10.1007/978-3-031-31967-9_2

[17] Ovsyannikov S., Kalinin E., Koliesnik I. (2020) Oscillation Process of Multi-support Machines When Driving Over Irregularities. In: Murgul V., Pasetti M. (eds) International Scientific Conference Energy Management of Municipal Facilities and Sustainable Energy Technologies EMMFT 2018. EMMFT-2018 2018. Advances in Intelligent Systems and Computing, vol 982. Springer, Cham. https://doi.org/10.1007/978-3-030-19756-8_28.

[18] Haniszewski T. Koncepcja projektu stanowiska do badania zjawisk dynamicznych zachodzących podczas unoszenia ładunku. Wydawnictwo Politechniki Śląskiej, Zeszyty Naukowe. Transport Politechnika Śląska. 2015. Z 88. S. 49-60.

[19] Iain A. Martin, Rishad A. Irani A generalized approach to anti-sway control for shipboard cranes. Mechanical Systems and Signal Processing 148 (2021) 107168 https://doi.org/10.1016/j.ymssp.2020.107168