CONTACT INTERACTION OF BODIES ALONG CONGRUENT SURFACES

Mykola M. Tkachuk[1], Olena Zinchenko[1], Andriy Grabovskiy[1], Mykola A. Tkachuk[1],
Volodymyr Sierykov[1], Natalia Domina[2], Iryna Hrechka[1]
[1]National Technical University «Kharkiv Polytechnic Institute», Kharkiv, Ukraine
[2] Dmytro Motornyi Tavria State Agrotechnological University, Zaporizhzhia, Ukraine
Email: andrey.grabovskiy@khpi.edu.ua

Abstract - The contact interaction of complex-profile elements of machine-building structures is in many cases a factor that determines the performance, durability, load capacity and other characteristics of the product as a whole. The paper describes a variational formulation of the problem of contact interaction of bodies. The purpose of the work is to improve the variational formulation of the problem of the contact interaction of elastic-plastic bodies under perturbances in the shape of nominally matching contact surfaces and the properties of the surface layers of the material. The contact of bodies along congruent surfaces is studied. In addition, the modification of these surfaces is considered. The problem is reduced to a variational inequality. The roughness of the surface layers of contacting bodies is taken into account. Plastic deformations of these bodies' materials are also taken into account. It was established that the distribution of contact pressure is significantly uneven. It was also found that by modifying the geometric shape of the contacting surfaces, it is possible to achieve a reduction in the level of contact pressure. In addition, a similar effect is observed when reducing the contact roughness of the surface layers of the materials of the contacting bodies. In the presence of a set of factors taken into account (disturbance of nominally coincident surfaces, influence of surface layers' roughness and elastic-plastic deformation of the materials of the contacting bodies), the resulting effect exceeds those of each of the effects. Therefore, by means of certain constructive and technological measures, it is possible to significantly improve the conditions of contact interaction between bodies.

Keywords: Contact interaction, Variational inequality, Contact roughness, Congruent surfaces, Surface modification.

1. Introduction

In many designs, the contact of parts along congruent (coinciding) surfaces is used to transmit power by individual parts. At the same time, there is a problem of determining the stress-strain state (STS) taking into account the contact interaction of these bodies. These are the so-called conformal contact problems [1-7]. Such tasks include, in particular: contact in splined joints, toothed gearings, in engine elements, in bearings, etc.

When analyzing the contact interaction with accounting for the specified features, it should be taken into consideration that in reality, contact along nominally matching smooth surfaces is not realized. This is caused by the presence of rough layers, films, coatings, etc. in the contact zone. In addition, there are inevitably macro deviations from the nominal shape of the contacting surfaces due to errors in manufacturing. Errors in installation, assembly and deformation of other parts (shafts, bearings, supports, etc.) on which the studied contacting bodies are installed also contribute to the problem.

Therefore, the application of models of conformal contact problems [1, 8-10] can lead to a distortion of the real picture of contact interaction.

One more factor should be taken into account. The fact is that in addition to unintentional disturbances of the nominal properties of the contacting bodies' surfaces, it is also possible to consider their intentional change. This is a modification of these surfaces. Such a modification is intended, firstly, to compensate for the negative impact of the unintentional disturbances listed above, and, secondly, to improve the conditions of the contact interaction of structural elements.

Another important factor that must be taken into account in many cases is the plastic deformation of the contacting bodies. Indeed, compared to the case of elastic bodies' contact, the appearance of plastic deformations can significantly affect the contact interaction of the studied bodies.

Therefore, as a result, a problem arises that requires development of traditional problem setting [8-10]. At the same time, it is necessary to take into account the above-mentioned factors: disturbance of nominally matching surfaces, influence of surface layers' roughness and elastic-plastic deformation of the contacting bodies' materials.

Such task is considered in this work.

2. Analysis of Existing Methods of Stress-strain State Analysis and Contact Interaction of Structural Elements

The mechanics of contact interaction [8-10] has significant achievements in its development.

In particular, the Hertz model for analyzing the contact interaction of smooth bodies should be noted. The gap between the bodies is given in the form of a sufficiently defined quadratic form of coordinates [10]. However, for the case considered in the paper, this model is not applicable.

Among the more universal, compared to the Hertz model, the theory of variational inequalities should be noted [11]. Thus, in the case of smooth bodies, the problem of their contact is reduced to the problem of minimizing the functional of the total energy of the elastic bodies system within a set of displacements distributions that satisfy the conditions of non-penetration of the points of the bodies into one another on the surfaces of possible contact. However, such a classic statement does not take into account the entire complex of factors that are important in the case under consideration (see above).

At the same time, the approach itself, which reduces the contact problem to a variational problem, is quite promising.

Among the set of problems on the contact interaction of structural elements, a number of them attract attention, which are similar in formulation to the researched ones.

For example, in work [12] it was established that stresses arising in torsion shafts are mainly concentrated in the zone of contact interaction of the splined shaft head with the splined coupling. Modelling of the elastic-plastic deformation of torsion shafts of vehicle suspension systems, taking into account the contact interaction, is reduced to a variational formulation of the problem based on the theory of variational inequalities. The method proposed in the paper is applied to calculate the stress-strain state of the torsion shaft, taking into account plastic deformation of the material and contact interaction with contacting bodies.

In work [13], to model the reaction of torsion shafts to the action of the torque, the stress-strain state is analyzed taking into account the contact interaction with the splined sleeve. The peculiarities of contact pressure distribution between these bodies have been established. The nature of the stress concentration in the splined depressions of the shaft head is determined. Research models and methods have been developed that make it possible to develop recommendations for design solutions when designing vehicle suspension systems.

The factors that determine the strength of the torsion shaft for the values of the diameters of the torsion shaft head, which are close to the diameters of the torsion shaft stem, are determined. In the case under consideration, it is primarily the strength of the torsion shaft's head. In particular, it was established that significant plastic deformations and contact loads occur in the torsion shaft heads in the course of manufacturing operations. This factor is decisive when justifying the design parameters of torsion shafts.

It is stated in this work, following reviewing [14-17], that there is an urgent task of developing improved approaches, models and methods of researching the stressed and deformed state of a torsion shaft, taking into account the elastic-plastic nature of the deformation in the entire body of the shaft, including its head, as well as taking into account the contact interaction in the splined joint. In the same works, it is mainly about the stem (cylindrical) part of the torsion shaft when determining the strength of the torsion shaft. The solution of such problem was carried out in work [13].

The contact interaction of complex-profile elements of machine-building structures is in many cases a factor that determines the performance, durability, load capacity and other characteristics of the product as a whole. Examples include rolling bearing bodies, gear teeth with modified working surfaces, ball pistons of radial hydraulic transmissions and many other complex-profile bodies.

In particular, the purpose of the study [18] is to increase the energy efficiency and resource of highly loaded teeth of the driving gear in transmission shafts of core drilling pumps due to the substantiation of critical loads and stresses in hardened gear coatings that operate under conditions of intensive wear of the contact surface with fracture. A simulation of gear wheels' contact demonstrates the conditions under which the maximum stress values occur, which can disturb the balance of forces and deform the geometry of the tooth.

Classical calculation methods do not take into account the influence of the intensity and volume of wear on the change in the structural geometry of the part. A worn contact surface affects the dynamics of force moments and contact stress redistribution and, as a result, changes the structure of the relative position of the contact surfaces relative to the calculated axis of symmetry [19].

When the material wears out, the width of the tooth changes, which leads to a change in the engagement angle and the contact surface area of the tooth [20].

In works [21-23], it is noted that as a result of mechanical wear, deviation of the angle of gear engagement and other structural elements, the action of the radial force changes significantly, along with the vector of its application per contact area unit.

Localization of zones of internal fatigue stress concentration is a problem of extending the resource. It was also noted that understanding the principle of redistribution of metal structure fatigue phenomena during operation will enable effective development of technical solutions for construction and manufacturing technology. Long-term operation brings slight surface wear, which contributes to the loose fit of the contacting surfaces and, as a result, a deviation of the calculated axes of the contact transmission (change in the design of the mechanism), which causes a redistribution of the load on the teeth surface [18].

A complex of studies [24-26] is devoted to the analysis of the stress-strain state of complex-profile elements of rolling bearings under conditions of materials' contact interaction and elastic-plastic deformation.

However, these works do not take into account the entire complex of significant factors.

In particular, many works [27-30] describe models of the properties of materials' surface layers (roughness, sputtering, films, coatings, etc.).

Another important factor is the model of elastic-plastic deformation of materials [31].

To date, there are no universal formulations for generating solutions for analysis of bodies' contact interaction with accounting for various factors. However, certain formulations in the works by Zhao J., E. Vollebregt C. [32] are adapted to development in areas of interest. Variational principles, which can be formed on the development of Kalker's principle [33], have a certain universality with respect to the properties of contacting bodies. This makes it possible, using Kalker's principle as the initial "core", to build it up due to the terms in which various characteristic properties of certain objects under study are concentrated. This makes it possible to take into account the nonlinear properties of the surface layers of body materials that are subjected to technological operations of strengthening, spraying, heat treatment, etc. The influence of these effects is significant; therefore, it is necessary to develop physically adequate and mathematically rigorous models that predict, in particular, the dependence of the behavior of the studied objects' materials on the load history.

However, up to date, there are no such developments in the literature, which are fully aimed at the listed problematic aspects.

At the same time, in [12, 28] models and methods of contact interaction research are described, which are open to expansion to new classes of problems.

In [28], theoretical foundations of stress-and-deformed state are formed, taking into account the contact interaction of complex-profiled bodies and substantiating the properties of non-traditional materials and the shape of the surfaces of the contacting bodies according to strength criteria. For the first time, nonlinear mathematical models of material behavior on the contact surface or intermediate layer were developed. They are built on the basis of a combination of models of contact of micro-uniformities and conditions of penetration of bodies into one another. These models, firstly, reflect more adequately in physical terms the mechanism of contact interaction of rough bodies, and, secondly, unlike the traditional linear conditions of kinematic contact, lead to more complex, but more accurate, nonlinear mathematical models.

This creates new opportunities for analyzing the contact interaction of bodies, as a new important factor is taken into account. The boundary elements method received further development in the direction of solving structurally-physically nonlinear problems of contact interaction, which contain nonlinear, rather than linear members, as in traditional approaches, under conditions of movements' compatibility at the boundaries of contacting bodies. For this purpose, a modification of Kalker's variational principle was developed for the case of physically nonlinear intermediate layers.

Thus, it is possible to take the variational formulation of the contact problem as a starting point, supplementing it with new significant factors: perturbation of the shape of the surfaces of contacting bodies, the properties of their surface layers, and elastic-plastic deformation of materials.

The purpose of the work is to improve the variational formulation of the problem of the contact interaction of elastic-plastic bodies under perturbances in the shape of nominally matching contact surfaces and the properties of the surface layers of the material.

Tasks of research:

1. Development of an improved variational formulation of the problem.
2. Creation of numerical models of the stress-strain state of contacting bodies.
3. Research of stress-strain state and contact interaction on the example of torsion shafts of vehicle suspension systems.

3. Development of an Improved Variational Formulation of the Problem

The contact of two bodies 1 and 2 occupying the area of Ω_1 and Ω_2 is considered. They contact along the surfaces S_1^C and S_2^C . In the nominal case:

$$S_1^C \equiv S_2^C , \tag{1}$$

that is, these surfaces are congruent (matching).

If we denote the distribution of displacements in these bodies as

$$u_1 = u_1(r_1),\ u_2 = u_2(r_2), \qquad (2)$$

where r_1, r_2 are the radius-vectors of the points of bodies Ω_1 and Ω_2, which are specified in the Cartesian coordinate system $Ox_1x_2x_3$, then the contact problem is reduced to the problem of minimizing the energy functional [34]

$$I = \frac{1}{2}\int_{(\Omega)} \sigma_{ij} \cdot \varepsilon_{ij} d\Omega - \int_{(S^F)} F \cdot u\, dS - \int_{(\Omega)} f \cdot u\, d\Omega\ . \qquad (3)$$

In (3),

$$\Omega = \Omega_1 \cup \Omega_2, \qquad (4)$$

$$S^F = S_1^F \cup S_2^F, \qquad (5)$$

where S_1^F, S_2^F are areas of the surfaces of bodies 1 and 2 loaded with distributed forces F; f – volumetrically distributed forces.

At the same time, in (3), the components of the stress σ and strain tensors ε depend on the of the displacement vector components $u = \{u_1, u_2, u_3\}^T$ according to the dependencies [34]

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \qquad (6)$$

$$\sigma_{ij} = C_{ijkl} \cdot \varepsilon_{kl}\ , \qquad (7)$$

where C_{ijkl} is the tensor of the modulus of elasticity of the materials of the contacting bodies.

Problem (3) takes the form:

$$u^* : I(u) \to \min\ , u \in K\ , \qquad (8)$$

where:

$$K\{u : u^1_{s_1^c} + u^2_{s_2^c} \le \delta_{12}\}\ . \qquad (9)$$

Here K is a set of displacements distributions along the normals ν_1 and ν_2, which satisfy the conditions of impenetrability of bodies to one another on the surfaces S_1^C, S_2^C respectively with a gap δ_{12} between them.

It is also assumed that the anchoring conditions also apply

$$u\big|_{S^u} = 0\ , \qquad (10)$$

where $S^u = S_1^u \cup S_2^u$ is a fixed surface connecting the fragments of bodies' 1 and 2 surfaces.

If we turn to problem (8), it should be noted that for the case of contact between elastic smooth bodies we have a quadratic programming problem [35]. Let's return to the development of problem (8), (9) for the case of accounting for new factors.

1). Elastic-plastic deformation of materials in contacting bodies.

In this case, the stress-strain state of contacting bodies depends, in contrast to the case of elastic bodies, on the load history [31, 34]. To take this dependency into account, the parameter τ responsible for the evolution of loads is introduced:

$$F = F(r, \tau)\ ;\ f = f(r, \tau)\ . \qquad (11)$$

Let us apply it to the analysis of the stress-strain state and the contact interaction of the incremental type dependency [34, 36]. They can be written in load increments dF, df, displacement vectors du, stress $d\sigma$ and strain tensors $d\varepsilon$. Then (6) and (7) are transformed to the form:

$$d\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial du_i}{\partial x_j} + \frac{\partial du_j}{\partial x_i}\right), \qquad (12)$$

$$d\sigma_{ij} = C_{ijkl}(\sigma, \varepsilon) \cdot d\varepsilon_{kl}\ , \qquad (13)$$

where C_{ijkl} tensor depends on the current stress-strain state and the type of load (active, neutral, or unloading) [34, 36].

Under these conditions, the structure of the functional (3) remains the same, but in displacements:

$$I(du) = \frac{1}{2}\int_{(\Omega)} d\sigma_{ij} \cdot d\varepsilon_{ij} d\Omega - \int_{(\Omega)} dF \cdot du\, d\Omega - \\ - \int_{(S^{//})} df \cdot du\, dS \qquad (14)$$

Then (8), (9) take the form:

$$I(du) \to \min\ , \qquad (15)$$

$$K\{u : u^1_{s_1^c} + u^2_{s_2^c} \le \tilde{\delta}_{12}(\tau)\}\ , \qquad (16)$$

where $\tilde{\delta}_{12}(\tau)$ is the current gap between the surfaces S_1^C and S_2^C.

Finally we have:

$$u = \int_{\tau} du \cdot d\tau\ . \qquad (17)$$

Thus, the original problem is reduced to a sequence of problems in increments, which structurally correspond to the traditional problem [37, 38], although with variable physical and mechanical properties of the bodies' materials and replacement of the initial gap with the current one.

2). Taking into account the properties of the surface layers of the contacting bodies' materials.

In order to take into account, the properties of the surface layers of the materials of the contacting bodies, they are presented in the form of some additional fictitious bodies that simulate the properties of these surface layers. Applying Winkler-type models [11, 31] to these layers, we have an additional addition to the functional dI:

$$I(du) := I(du) + \frac{1}{2} \int_{(S^c)} c^2 \cdot d\Delta_{12}\,, \tag{18}$$

where $c = c(u)$ is the current roughness of the materials' surface layers, Δ_{12} is the value of the normal compression of these layers.

Dependencies $c(u, \tau)$ determine physical and mechanical properties of the surface layers' materials. They depend on the structure of these layers (technological modes, structure and composition of coatings, etc.), as well as on the current stress-strain state.

At the same time, Δ_{12} depends linearly on the increments components du. Therefore, the functional (18) remains structurally the same as (14).

3). Taking into account the disturbance of the geometric shape of the contacting bodies' surfaces.

When the surfaces of contacting bodies are disturbed, two effects occur.

I. The gap between contacting bodies changes:

$$\delta_{12} := \delta_{12} + \Delta\delta_{12}\,, \tag{19}$$

where $\Delta\delta_{12}$ is the gap perturbation.

II. Areas Ω_1 and Ω_2 also change due to disturbances:

$$\Omega_1 := \Omega_1 - \Delta\Omega_1\,,\ \Omega_2 := \Omega_2 - \Delta\Omega_2\,. \tag{20}$$

Due to additivity of the integrals in (13) and considering the smallness of the areas' perturbations of the compared to the areas themselves, such a perturbation of the functional can be modelled by fictitious physical and mechanical properties of materials. That is, in the perturbation areas, the tensor components C_{ijkl} become zero.

Summing up, it can be noted that based on the development of [12, 13, 28, 31], a methodology for taking into account new additional factors for the analysis of the stress-strain state and the contact interaction of bodies along matching (congruent) surfaces has been proposed. At the same time, dependence of the stress-strain state and contact interaction on the load history is considered. The individual case of the contacting surfaces' congruence, in particular, leads to the fact that the initial gap

$$\delta_{12}(\tau = 0) \equiv 0\,. \tag{21}$$

The described models and methods are the essence of the developed more universal approach to the analysis of contact interaction of elastic bodies.

4. Creation of Numerical Models of the Stress-strain State of Contacting Bodies

To analyze the stress-strain state and the contact interaction of bodies with nominally matching contacting surfaces, the case of the interaction of a fragment of a torsion shaft of a vehicle suspension system with a spline coupling is considered. In fig. 1 the geometric model of these rotation bodies is shown. Due to cyclometry, the sector of $360^\circ/39 = 9{,}23^\circ$ is highlighted.

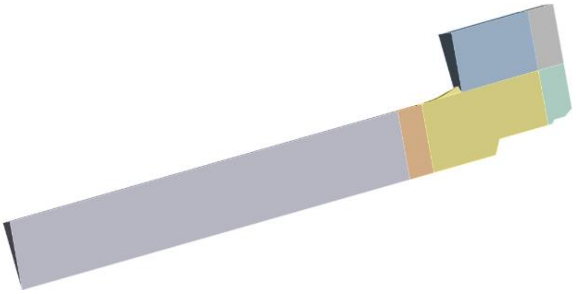


Figure 1: Geometrical model of the torsion shaft (1/39th part) of a vehicle's suspension system with a splined coupling

In fig. 2, marginal conditions are shown. In fig. 3 – load conditions. Here $\tau \in [0;2]$ is the load parameter. At the same time, the load is carried out by turning the right cross-section of the torsion shaft to the twist angle: $\gamma(0) = 0^\circ; \gamma(1) = 5^\circ; \gamma(2) = 0^\circ$.

The following are being investigated:

A. The case of nominally congruent surfaces and elastic behavior of the contacting bodies' material.

Properties of the bodies' materials: modulus of elasticity E=200 GPa, Poisson's ratio ν=0.3.

B. The case of nominally congruent surfaces and elastic-plastic deformation of bodies' materials.

Material properties of elastic-plastic bodies' materials: yield strength – 1600 MPa, cross-sectional elasticity modulus – 3200 MPa.

C. The case of perturbation of the geometric shape of contacting bodies and the elastic behavior of the materials of the bodies.

In fig. 4, the form of disturbance in the axial section is demonstrated.

D. The case of perturbation of the geometric shape of the contacting bodies and the elastic-plastic behavior of the materials of the bodies.

E. The case of perturbation of the geometric shape of the contacting bodies and the elastic surface layer.

F. The case of disturbance of the geometric shape of the contacting bodies, the elastic-plastic behavior of the materials of the bodies and the presence of an elastic surface layer.

Study of stress-strain state and contact interaction on the example of torsion shafts of vehicle suspension systems. In fig. 5–10 are the results of studies of the stress-strain state and contact interaction of torsion shafts of vehicle suspension systems.

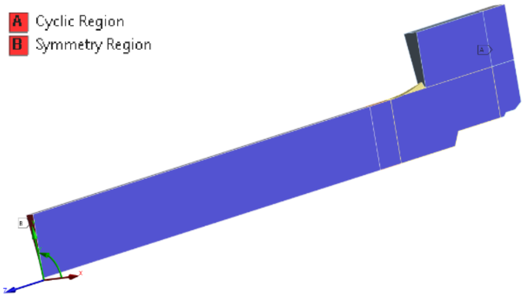
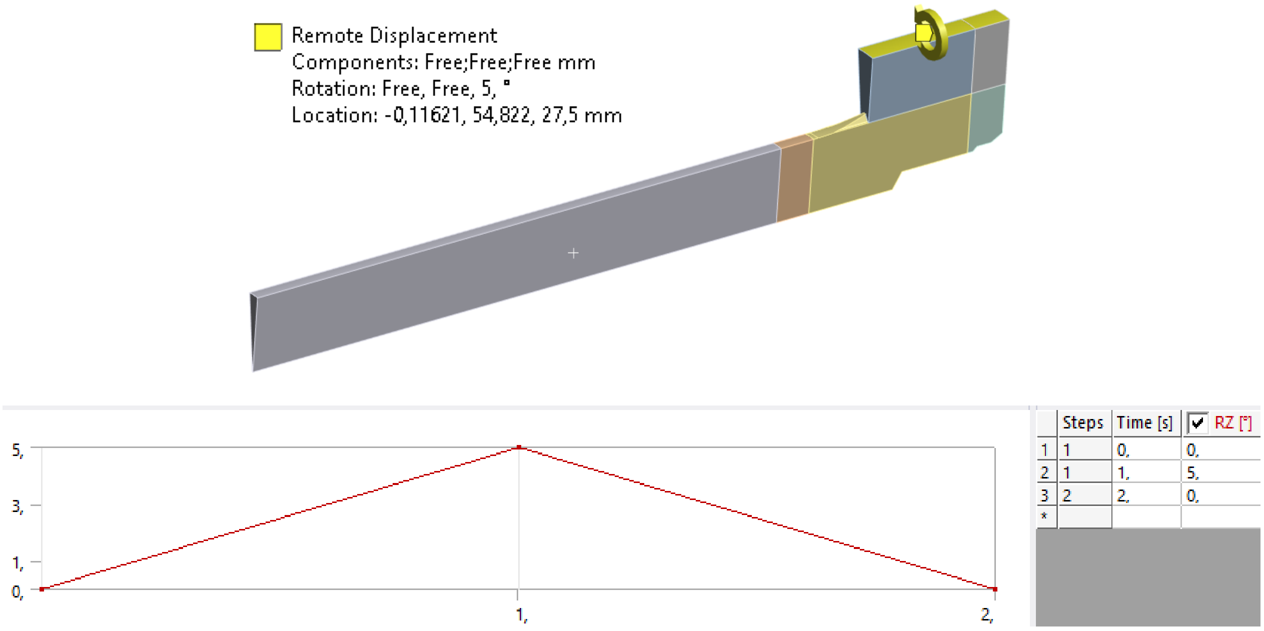


Figure 2: Boundary conditions for contacting bodies


Remote Displacement
Components: Free;Free;Free mm
Rotation: Free, Free, 5, °
Location: -0,11621, 54,822, 27,5 mm

	Steps	Time [s]	☑ RZ [°]
1	1	0,	0,
2	1	1,	5,
3	2	2,	0,
*			

Figure 3: Load conditions for contacting bodies

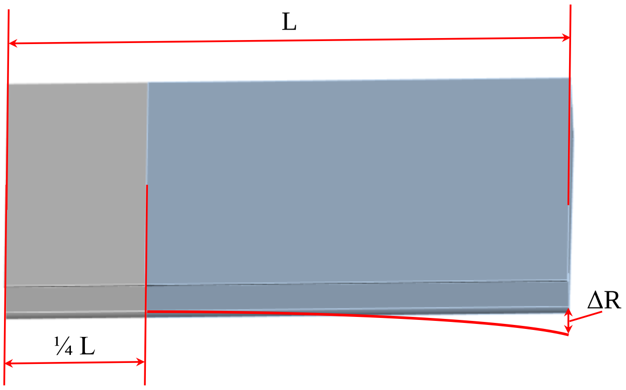


Figure 4: Form of disturbance of contacting bodies

As can be seen from the analysis of the results of the conducted research, the following features are inherent in the stress-strain state and contact interaction.

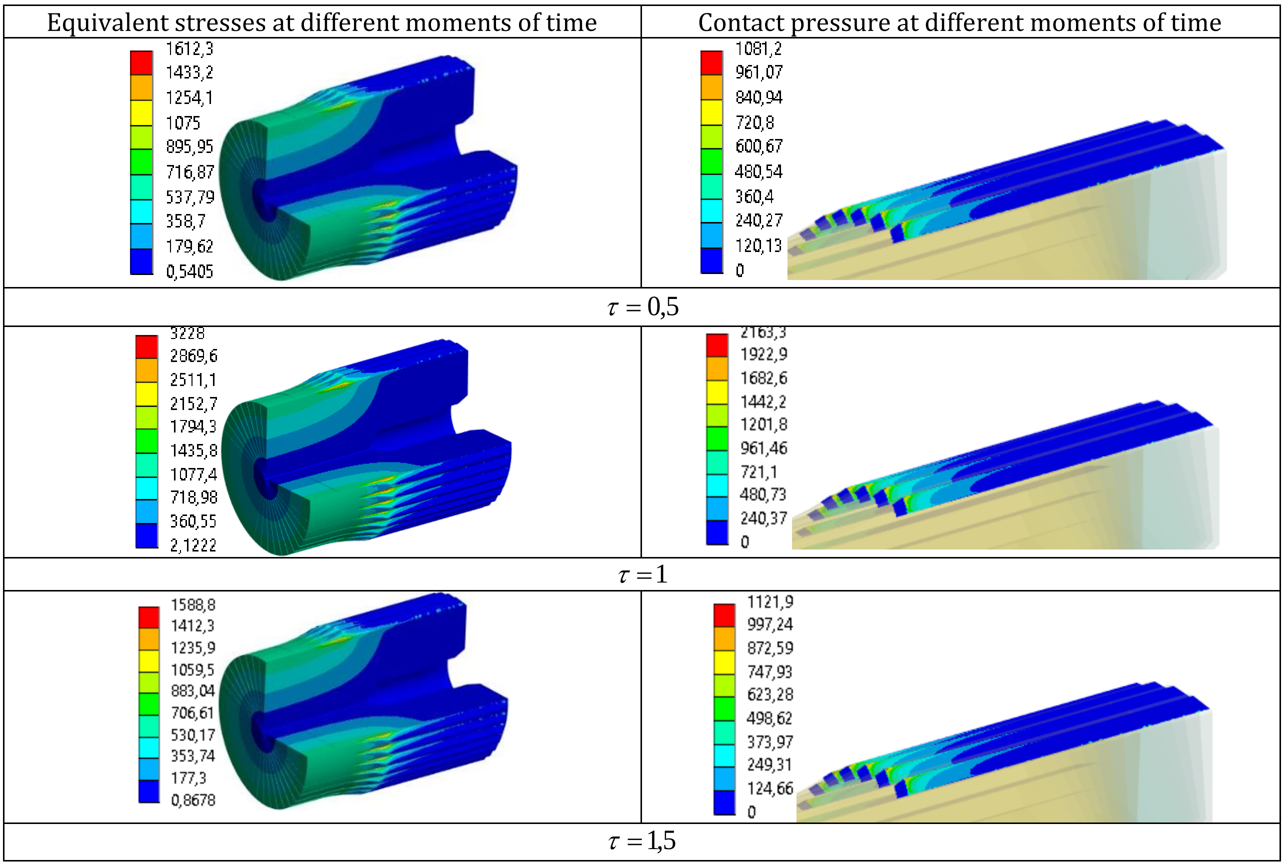


Figure 5: Results of studies of the stress-strain state and contact interaction of torsion shafts of vehicle suspension systems: equivalent Mises stresses and contact pressure in the torsion head (option A)

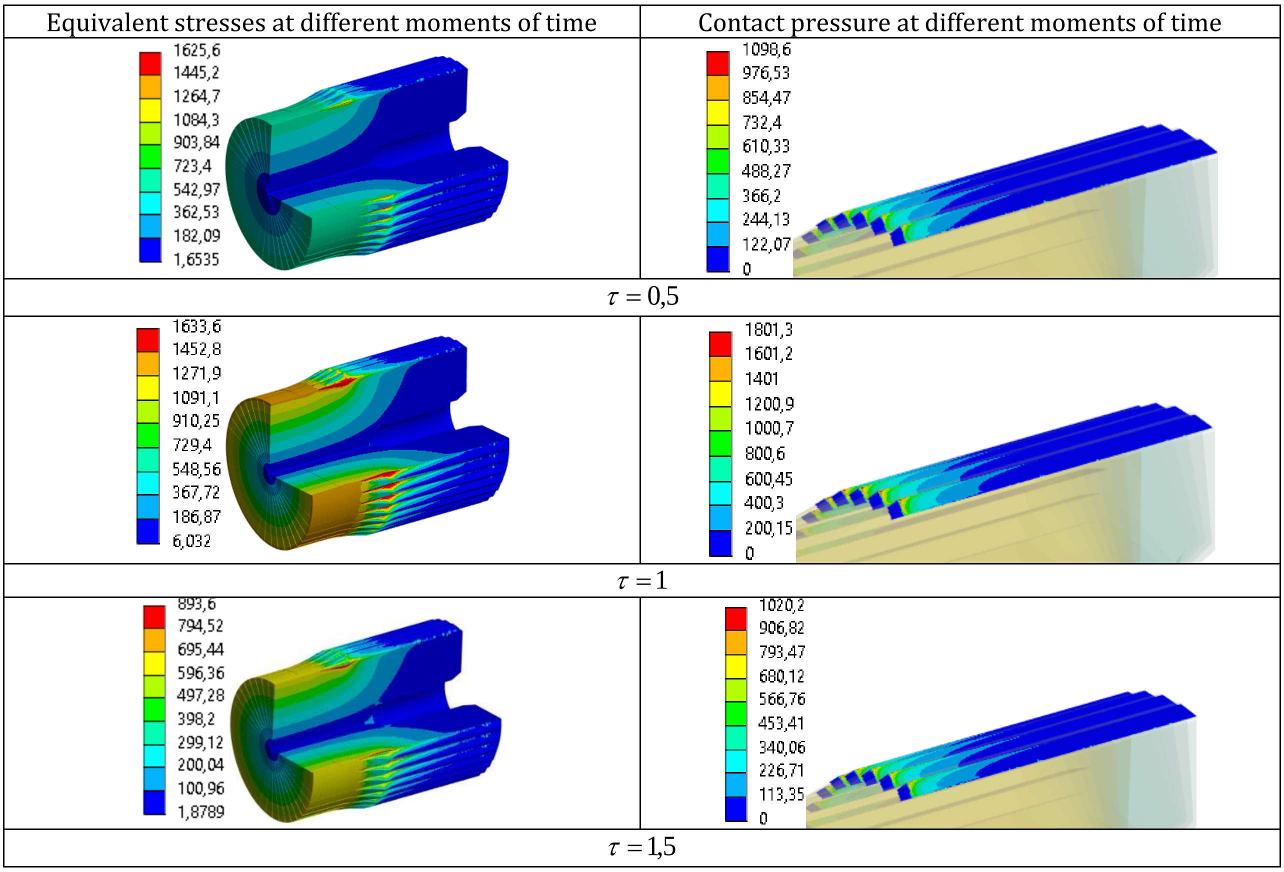


Figure 6: Results of studies of the stress-strain state and contact interaction of torsion shafts of vehicle suspension systems: equivalent Mises stresses and contact pressure in the torsion head (option B)

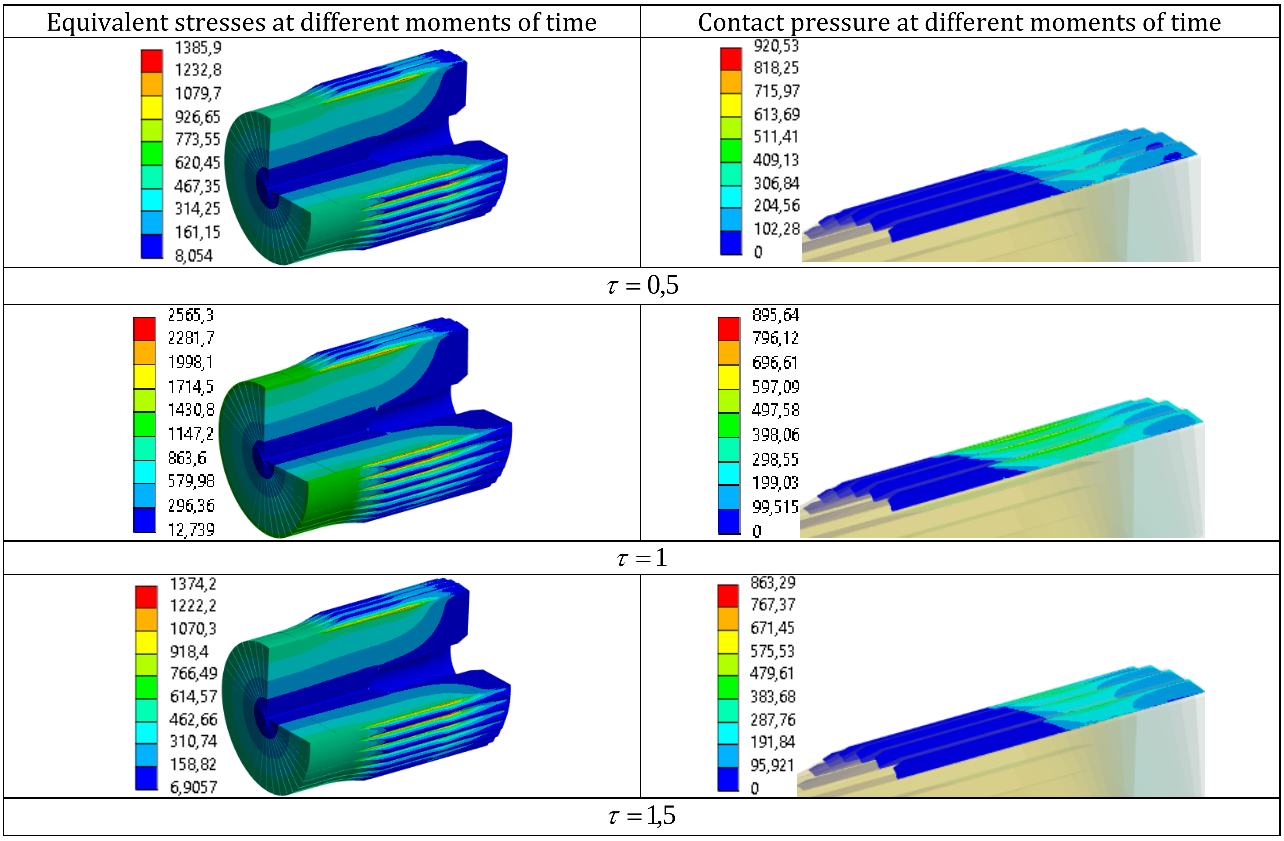


Figure 7: Results of studies of the stress-strain state and contact interaction of torsion shafts of vehicle suspension systems: equivalent Mises stresses and contact pressure in the torsion head (option C)

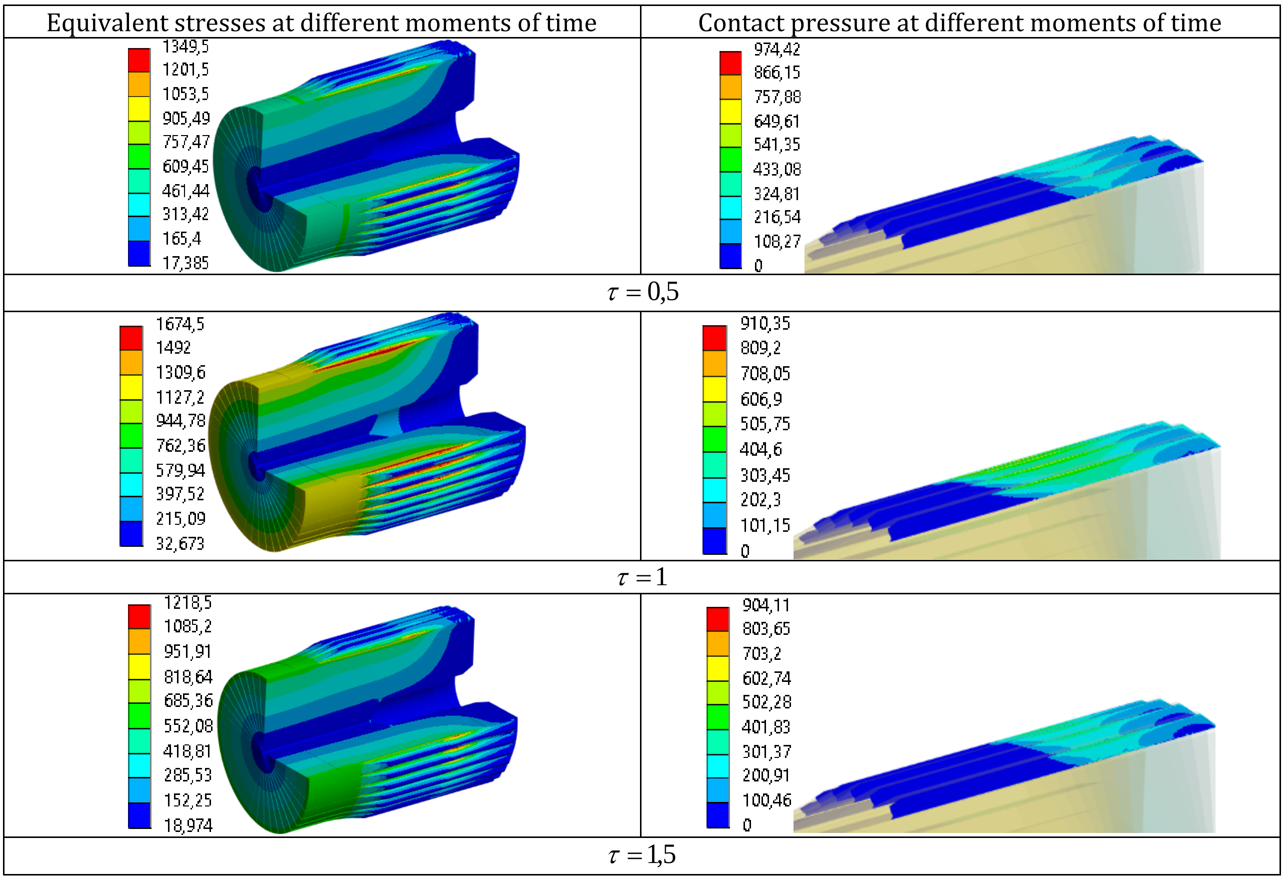


Figure 8: Results of studies of the stress-strain state and contact interaction of torsion shafts of vehicle suspension systems: equivalent Mises stresses and contact pressure in the torsion head (option D)

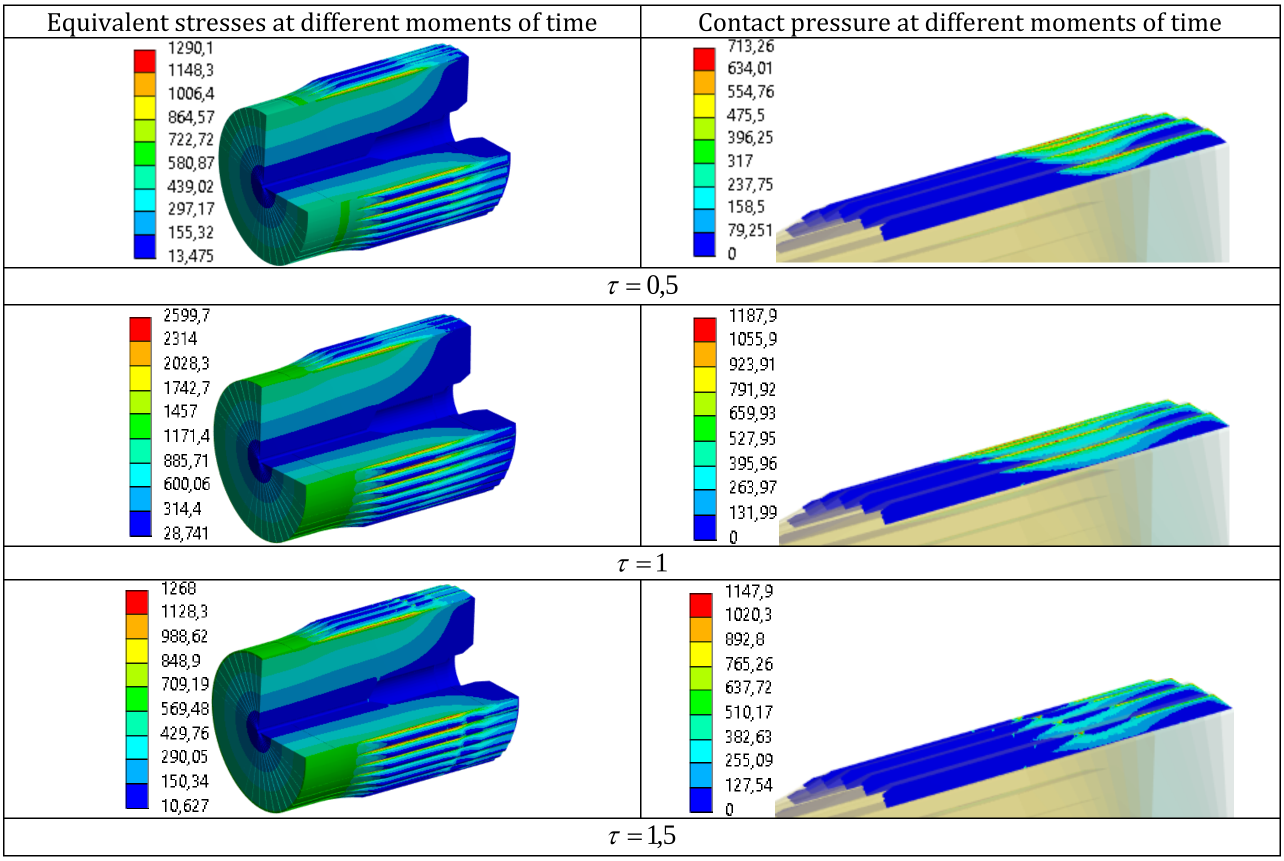


Figure 9: Results of studies of the stress-strain state and contact interaction of torsion shafts of vehicle suspension systems: equivalent Mises stresses and contact pressure in the torsion head (option E)

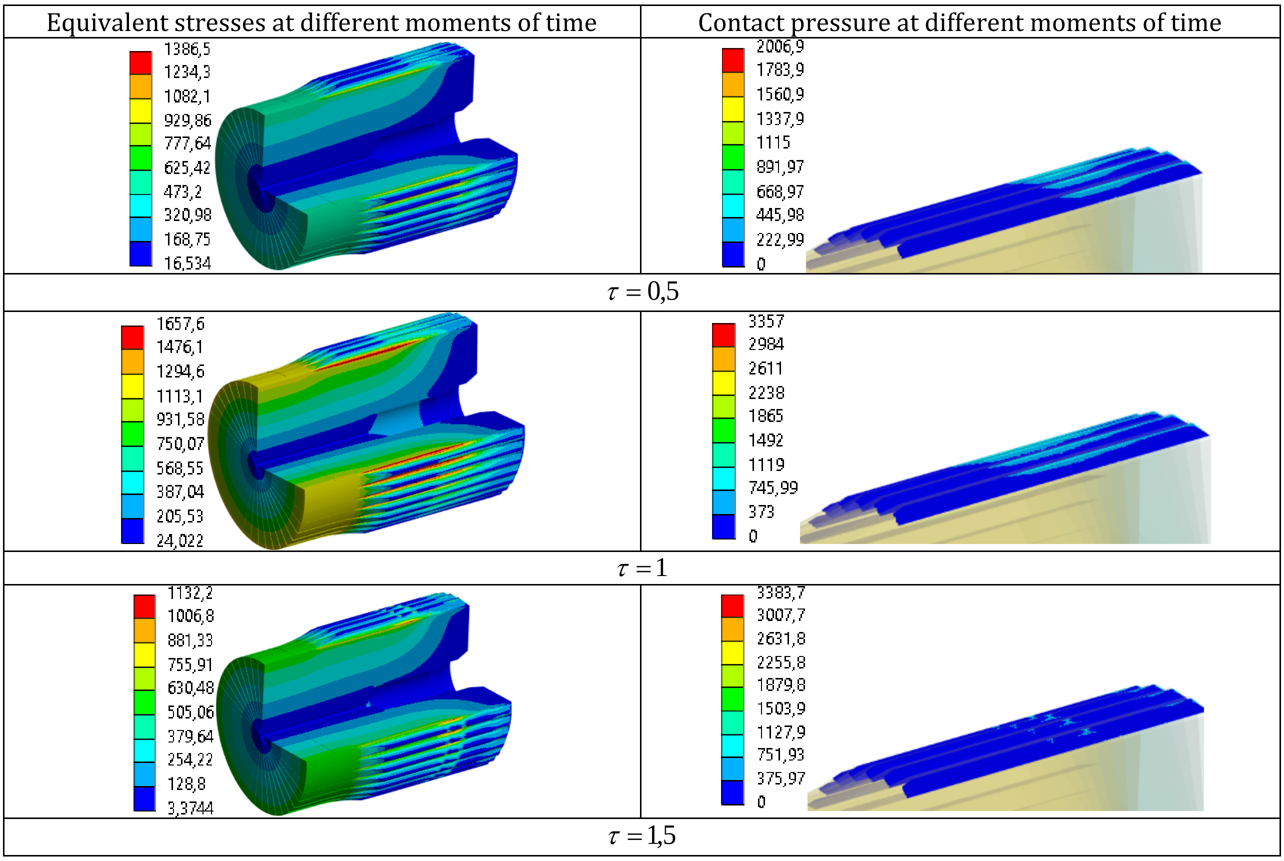


Figure 10: Results of studies of the stress-strain state and contact interaction of torsion shafts of vehicle suspension systems: equivalent Mises stresses and contact pressure in the torsion head (option F)

5. Conclusions

1. Case A is characterized by a high unevenness of the contact pressure distribution between the torsion shaft and spline coupling.

2. Taking into account elastic-plastic deformation leads to a more uniform distribution of contact pressure and a decrease in its level compared to the case of accounting for elastic deformations only.

3. A certain distortion of the shape of the torsion shaft's surface, taking into account the stiffness of the surface layers of the materials of the contacting bodies also lead to a decrease in the level of contact pressure and a more uniform distribution of it.

4. In the presence of a set of factors that were taken into account, the resulting effect exceeds those of the action of each individual effect.

Thus, we can draw a general conclusion that by introducing perturbations to the geometric shape of bodies contacting along nominally congruent surfaces and properties of materials' surface layers, as well as changes in physical and mechanical properties during elastic-plastic deformation of materials, it becomes possible to significantly reduce the level of contact pressure. Therefore, by means of certain constructive and technological measures, it is possible to significantly improve the conditions of contact interaction of these bodies.

The research was carried out as part of the EURIZON N3055 project "Combined technologies of metallic surface modification by micro-arc oxidation and boriding for critical machine parts with high contact loads".

References

[1] Dmytriv K. M., Dyachenko N. M. Porivnyal'nyy analiz konformnykh ploskykh kontaktnykh zadach pro znoshuvannya za stepenevym zakonom pry fiksovaniy ploshchadtsi kontaktu. [Comparative analysis of conformal contact problems of wearing under the power law with fixed contact zone] *Computer Science and Applied Mathematics.* № 2. (2021). Pp. 14-24. DOI: https://doi.org/10.26661/2413-6549-2021-2-02 [in Ukrainian].

[2] Han, D., Han, W., Jureczka, M., Ochal, A. (2020) Numerical analysis of a contact problem with wear. *Computers & Mathematics with Applications,* vol. 79, pp. 2942 – 2951. https://doi.org/10.1016/j. camwa.2019.12.027.

[3] Jureczka M., Ochal A. (2019) Numerical analysis and simulations of contact problem with wear. *Comput. Math. Appl*., vol. 77, pp. 2980–2988. https://doi.org/10.1016/j.camwa.2018.08.044

[4] Xuan H., Cheng X. (2020) Numerical Analysis and Simulation of a Frictional Contact Problem with Wear, Damage and Long Memory. *East Asian Journal on Applied Mathematics,* vol. 10, no. 4, pp. 659–678. https://doi.org/10.4208/eajam.130320.260520

[5] Voitov A.V. (2020) Modeling the processes of friction and wear under dynamic influences on the tribosystem. *Problems of trbology*, vol. 25, no. 3/97, pp. 45 –49. https://doi.org/10.31891/2079-1372-2020-97-3- 45-49.

[6] Savchuk Yu. V, Maksimuk O.V., Salanda I.P. (2019) Analitiko-chislove obchislennya iteracijnimi metodami oblastej kontaktu dlya zadachi pro znoshuvannya pruzhnoї pivploshchini kanonichnimi shtampami. [Analytical and numerical calculations by iteration method for contact area for the problem of wearing the elastic area by canonical stamps] *Mathematical and computer modeling. Series: Phys.-Math. Science*, issue 20, pp. 70 –78. https://doi.org/10.32626/2308-5878.2019-20.70-78. [in Ukrainian].

[7] Krol O, Sokolov V. Research of toothed belt transmission with arched teeth. *Diagnostyka*. 2020, 21(4), Pp. 15-22. https://doi.org/10.29354/diag/127193

[8] Barber J. R., Ciavarella M. Contact mechanics. *International Journal of solids and structures*. 2000. T. 37. №. 1-2. Pp. 29 – 43. DOI:10.1016/S0020-7683(99)00075-X

[9] Barber J. R. *Contact mechanics*. Springer, 2018. 212p. DOI:10.1016/S0020-7683(99)00075-X

[10] Johnson K.L. *Contact mechanics.* Cambridge university press. 1985. 462 p. doi:10.1017/cbo9781139171731

[11] Martynyak R.M, Prokopyshyn I.A, Prokopyshyn I.I. Contact of elastic bodies with nonlinear Winkler surface layers. *Journal of Mathematical Sciences.* Springer US. 2015. *T.* 205, vol. 4, pp. 535-553. DOI:10.1007/s10958-015-2265-0

[12] M. M. Tkachuk, A. V. Hrabovs'kyy, A. V. Zavorotniy, S. V. Kutsenko, M. S. Savers'ka, I. YE. Klochkov, O. I. Zinchenko, M. A. Tkachuk, S. O. Nazarenko, N. V. Pinchuk, S. I. Marusenko. Chysel'ne modelyuvannya pruzhno-plastychnoho deformuvannya torsionnykh valiv system pidresoryuvannya transportnykh zasobiv iz urakhuvannyam kontaktnoyi vzayemodiyi. [Numerical simulation of elastic-plastic deformation of torsion shafts of vehicle spring spring systems, taking into account contact interaction] *Visnyk Natsional'noho tekhnichnoho universytetu «KHPI». Seriya: Mashynoznavstvo ta SAPR.* № 1. 2022. Pp. 91-114. doi: 10.20998/2079-0775.2022.1.10 [in Ukrainian].

[13] A. V. Hrabovs'kyy, M. M. Tkachuk, A. V. Zavorotniy, S. V. Kutsenko, M. S. Savers'ka, I. YE. Klochkov, M. A. Tkachuk, O. I. Zinchenko, R. P. Derevyankin. Kontaktna vzayemodiya

torsionnoho vala iz shlitsevoyu vtulkoyu pry pruzhno-plastychnykh deformatsiyakh. [Contact interaction of a torsion shaft with a splined bush in elastically plastic deformations] *Visnyk Natsional'noho tekhnichnoho universytetu «KHPI». Seriya: Mashynoznavstvo ta SAPR.* № 1. 2021. Pp. 34-46. doi: 10.20998/2079-0775.2021.1.04 [in Ukrainian].

[14] V. Močilnik, Nenad Gubeljak, Jožef Predan. Model for fatigue lifetime prediction of torsion bars subjected to plastic presetting. *Tehnicki Vjesnik*. 2011.18(4):537-546. https://www.academia.edu/116075943/Model_for_Fatigue_Lifetime_Prediction_of_Torsion_Bars_Subjected_to_Plastic_Presetting

[15] V. Močilnik, Nenad Gubeljak, Jožef Predan. Surface residual stresses induced by Torsional Plastic Pre-setting of Solid Spring Bar. *International Journal of Mechanical Sciences.* 92 (2015). Pp. 269-278. https://doi.org/10.1016/j.ijmecsci.2015.01.004

[16] SUN Wei-qun, HE Feng-ming. Design and manufacture of vehicle torsion bar spring. *J. Automobile Science and Technolgy*. 2002, 2(2):23-27. https://caod.oriprobe.com/articles/4477697/Design_and_manufacture_of_vehicle_torsion_bar_spring.htm

[17] G. Zhou, X. Li, Y. Shi, Y. Zhu, B. Xu Numerical simulation of the working process and life prediction of a tank torsion shaft. *Journal of Mechanical Strength*. 2006, 28(4): pp. 578–581. DOI :10.16579/j .issn.1001.9669.2006.04.022

[18] Savinkin V. V. et al. Investigation of the Strength Parameters of Drilling Pumps during the Formation of Contact Stresses in Gears. *Appl. Sci.* 2021, *11*(15), 7076. https://doi.org/10.3390/app11157076

[19] Kyznetsova V.N., Savinkin V.V., Ratushnaya T.Y., Sandu A.V., Vizureanu P. Study of the spatial distribution of forces and stresses on wear surfaces at optimization of the excavating part of an earthmoving machine transverse profile. *Coatings.* 2021, 11, 182. Pp. 1-16. https://doi.org/10.3390/coatings110201821–16.

[20] Kadhimm Zarzoor A., Almuramady N., Hussein E.S. Stress analysis for spur gears using solid works simulation. *Int. J. Mech. Eng. Technol.* 2018, 259, 927–936. http://www.iaeme.com/IJMET/index.asp

[21] Sergienko, N., Medvediev, N., Grabovskiy, A., at all. Load and wear of the open hinge of the tractor caterpillar drive. *International Journal of Mechatronics and Applied Mechanics*, 2022, № 12, Pp. 12-20. https://ijomam.com/wp-content/uploads/2022/11/pp12-20.pdf

[22] Savinkin, V.V.; Kolisnichenko, S.N.; Sandu, A.V.; Ivanova, O.V.; Vizureanu, P.; Zhumekenova, Z.Z. Investigation of the Strength Parameters of Drilling Pumps during the Formation of Contact Stresses in Gears. *Appl. Sci.* 2021, *11*, 7076. https://doi.org/10.3390/app11157076

[23] Xiaofeng Q.; Jie L.; Xingguo Z; Li F.; Ruiqiang P. Fracture failure analysis of transmission gear shaft in a bidirectional gear pump. *Eng. Fail. Anal. Mach. Reliab.* 2020, 118, 138–141. https://doi.org/10.1016/j.engfailanal.2020.104886

[24] Zheng H., Tian G., Liang C., Zheng Y., Li G. Research on allowable contact stress of turntable bearing materials based on elastoplastic finite element method. Bearing, 2016 (01): 36-39.

[25] Deng B. et al. Sub-surface stress analysis of slewing bearing ball based on plastic deformation //IOP Conference Series: Materials Science and Engineering. IOP Publishing, 2020. T. 768. №. 4. P. 042027. DOI:10.1088/1757-899X/768/4/042027

[26] Zhang Qi, Lei Liangyu, Zhu Changcheng. Simulation analysis of wheel bearing rigidity based on ANSYS Workbench. *Journal of Xiamen University of Technology*, 2019, 27 (3): 37-42.

[27] Persson B.N.J. Elastoplastic contact between randomly rough surfaces. *Physical Review Letters*. 2001, vol. 87(11), pp. 116101. https://doi.org/10.1103/PhysRevLett.87.116101

[28] Tkachuk M. M. Mikromekhanichni modeli ta metody oserednennya vlastyvostey materialiv merezhevoyi struktury ta promizhnykh shariv kontaktuyuchykh til. [Micromechanical models and averaging methods for properties of materials with network microstructure and interface layers between contacting bodies] Diss... for obtaining the scientific degree of Doctor of Technical Sciences in the specialty 01.02.04 - mechanics of deformable solids. Institute of Mechanical Engineering Problems named after A.M. Pidgorny National Academy of Sciences of Ukraine, Kharkiv, 2019. 445 p. https://ipmach.kharkov.ua/wp-content/uploads/2020/01/Avtoref_TkachukMM_publish.pdf [in Ukrainian].

[29] Persson B.N.J. Relation between interfacial separation and load: a general theory of contact mechanics. *Physical review letters*. 2007. Vol. 99(12). P. 125502. https://doi.org/10.1103/PhysRevLett.99.125502

[30] Persson B.N.J, F. Bucher, Bernardino Chiaia Elastic contact between randomly rough surfaces: comparison of theory with numerical results. *Physical Review B*. 2002, vol. 65(18), pp. 184106. https://doi.org/10.1103/PhysRevB.65.184106

[31] Tkachuk M.M., Skripchenko N., Tkachuk M.A., Grabovskiy A. Numerical Methods for Contact Analysis of Complex-Shaped Bodies with Account for Non-Linear Interface Layers. *Eastern-European Journal of Enterprise Technologies*. 5/7 (95). 2018. Pp. 22-31. DOI: 10.15587/1729-4061.2018.143193.

[32] Zhao J., E. Vollebregt, C. Oosterlee. Extending the

BEM for elastic contact problems beyond the half-space approach. *Mathematical Modelling and Analysis*. 2016, 21(1), pp.119–141. https://doi.org/10.3846/13926292.2016.1138418

[33] Kalker J.J. Variational principles of contact elastostatics. *J. Inst. Math. And Appl.* 1977. Vol. 20. Pp.199-221. https://doi.org/10.1093/imamat/20.2.199

[34] K. Washizu. *Variational Methods in Elasticity and Plasticity Subsequent Edition*. Pergamon Pr; Subsequent edition. 1982. 630 p. https://doi.org/10.1002/zamm.19840640121

[35] Zhyl'tsov O. B. *Matematychne prohramuvannya (z elementamy informatsiynykh tekhnolohiy): Navch. posib. dlya stud. vyshch. navch. zakl.* [*Mathematical programming (with elements of information technologies): Education. manual for students higher education closing*] / O. B. Zhyl'tsov, V. R. Kulyan, O. O. Yun'kova; Za red. O. O. Yun'kovoyi. 2-he vyd., stereotyp. K.: DP «Vydavnychyy dim «Personal», 2008. 184 p. https://maup.com.ua/assets/files/lib/book/p10_11_ [in Ukrainian].

[36] Rudakov K.M. *Chyslovi i analitychni metody analizu dynamiky i mitsnosti mashyn ta stiykosti rukhu: Posibnyk*. [Numerical and analytical methods of analysis of dynamics and strength of machines and stability of motion]: Manual. K.: NTUU "KPI named after Igor Sikorskyi", 2022. 120 p.] https://ela.kpi.ua/bitstreams/2d01a788-7528-423a-a7b4-3e90e44dd5c6/download [in Ukrainian]

[37] Basova, Y., Dobrotvorskiy, S., Balog, M., Iakovets, A., Amine, C.M., Zinchenko, A. Increasing sme supply chain resilience in the face of rapidly changing demand with 3D model visualization, *International Journal of Mechatronics and Applied Mechanics*, 2023, 14, Pp. 35-47. https://doi.org/10.17683/ijomam/issue14.5

[38] Krol O., Sokolov V. *3D Modeling of Machine Tools for Designers. Monograph.* Sofia: Prof. Marin Drinov Academy Publishing House of Bulgarian Academy of Sciences, 2018. 140 p. https://doi.org/10.7546/3d_momtfd.2018