CONSERVED-QUANTITY-PRESERVING ALGORITHMS FOR DELTA PARALLEL ROBOT DYNAMICS

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Abstract - In order to overcome the non-geometric structure-preserving algorithm for the traditional numerical simulation of Delta parallel robot dynamics, this article investigates the Lie symmetries and conserved quantities of Delta parallel robot dynamics with three degrees of freedom under geometric constraints, as well as the conserved-quantity-preserving numerical methods. Firstly, considering the kinematic closed chain constraints, the Lagrange equations of the Delta parallel robot dynamics was established by using analytical mechanics method; Secondly, by introducing the theory of group analysis, the conditions for the existence of Lie symmetries in system dynamics and the corresponding forms of conserved quantities are given based on the principle of invariance. Finally, by transitioning the Lagrange equations to the Hamiltonian systems, a conserved-quantity-preserving discrete numerical algorithm was constructed for the regular equations of Delta parallel robot dynamics. The numerical simulation results show that using Lie symmetry theory to study the mechanical characteristics of the nonlinear dynamic system of Delta parallel robot, it can maintain the inherent structural properties of the system for a long time, with high computational efficiency, wide applicability, and small relative errors.

Keywords: Delta parallel robot, Lagrange equations, Lie symmetries, Conserved quantities, Conserved-quantity-preserving.

1. Introduction

The translational (3T) Delta parallel robot [1] with three degrees of freedom has high practical value in industries such as food, medicine, and electronics due to its simple structure and uncomplicated control. For Delta parallel robot, its dynamic characteristics are directly related to the relationship between robot joint torque and robot joint position, velocity, and acceleration. Therefore, the efficient dynamic calculation of robot is crucial for the structural optimization and real-time control [2-3]. At present, commonly used Delta robot dynamics modeling methods include virtual work principle method, Newton-Euler method, Kane method, and Lagrange method [4-7]. Among them, the Lagrange method for solving the dynamics of robot systems is a widely applicable analysis method, which avoids constraints and is described in matrix form, it is suitable for computer programming and can meet the real-time control requirement. However, the existing research results on the Lagrange method for the dynamics of 3T Delta robot are still insufficient.

Domestic and foreign scholars mainly focus on using various numerical calculation methods to study the dynamics of Delta robots. Whether it is software simulation or programming or calculating by user, the dynamic equations are discretized to simulate the dynamic response of the system under different parameter levels, and further discover the bifurcation, chaos, instability, and bullet-proof vibration of dynamic behavior [8]. The traditional approximate analytical methods [9] for nonlinear dynamic equations mainly involve asymptotic expansion of small parameter power, such as perturbation methods and multiscale methods, which are suitable for weakly nonlinear systems. When it comes to strongly nonlinear systems, although the incremental harmonic balance method and homotopy analysis can achieve certain results, overall, their calculation steps are complex and have significant limitations. The symmetry theory of dynamic systems is a higher-level rule in disciplines such as theoretical physics, engineering mathematics, and modern mechanics. The study of the symmetries of system motion equations can help to reveal the inherent characteristics and deep-seated laws of mechanical systems [10-11]. The symmetries and conserved quantities of mechanical systems are closely related, and the conserved quantities of the system not only has obvious physical significance, but also, they are the first integrals of the system, which can reduce the order of the equations [12]. However, the research on dynamic systems using symmetry theory both domestically and internationally has focused on general abstract mechanical systems [13], it leads to detachment from specific engineering applications, and its practical value has always needed to be further expanded. Fu [14] and Zheng [15] in China were the first to apply symmetry tool to some specific mechanical engineering cases. At the same time, there are few reports on using Lie symmetry methods to study the nonlinear dynamics of Delta robots both domestically and internationally. This article solves the Lie symmetries and conserved quantities of Delta robot, and constructs a conserved-quantity-preserving numerical difference algorithm, which can provide reference for the structural optimization design, motion trajectory control, and vibration prevention strategies of Delta robotic arms.

2. Materials and Methods

This paper deals with the phenomenon that the traditional numerical simulation algorithms of Delta parallel robot dynamics, which cannot guarantee the geometric structure of the system. In order to accurately and efficiently calculate the dynamic response of Delta parallel robot, the dynamic model of the Delta parallel robot is established according to the Lagrange equations. Further, using the Lie symmetry theory, the conserved quantities of Delta parallel robot is obtained. Most importantly, we have constructed a conserved-quantity-preserving numerical algorithm by using the canonical equations of system geometric structure. The dynamic response law of Delta parallel robot is obtained through conserved-quantity-preserving numerical algorithm.

3. Results and discussion

Research has shown that under the constraints conditions, the direction of optimizing the dynamic characteristics of Delta parallel robot is to increase the radius of the static and moving platforms, as well as the length of the slave arm, while reducing the size of the active arm. Meanwhile, comparing with existing literatures, the conserved-quantity-preserving numerical algorithms can simultaneously preserve constraints and conservative constants, and it can be stable for different step sizes. The system energy deviation does not diffuse, and the long-term simulation process of the system is not distorted.

3.1 Dynamic Model of Delta Parallel Robot

The three-dimensional structure of Delta robot studied in this article is shown in Figure 1. The robot is mainly composed of a frame, an active arm, a slave arm, a rotating shaft, a moving platform, and a gripper [16]. The retractable rotating shaft is used to drive the rotation of the gripper.

The static platform and the moving platform are connected by three axisymmetric motion chains, in which one active arm and one slave arm forming a motion chain. The three active arms are connected to the static platform through three rotating pairs. The active arm and the slave arm are connected in series by a pair of spherical pairs to form a motion chain. The slave arm consists of a closed-loop parallelogram structure consisting of two parallel rods and four spherical pairs. Finally, the remaining two spherical pairs on the slave arm connect the moving platform. The servo motor installed on the static platform drives the active arm to swing repeatedly, and the moving platform can achieve translational motion along the X, Y, and Z directions, thereby driving various coupled movements of the end effector gripper of the mechanism.

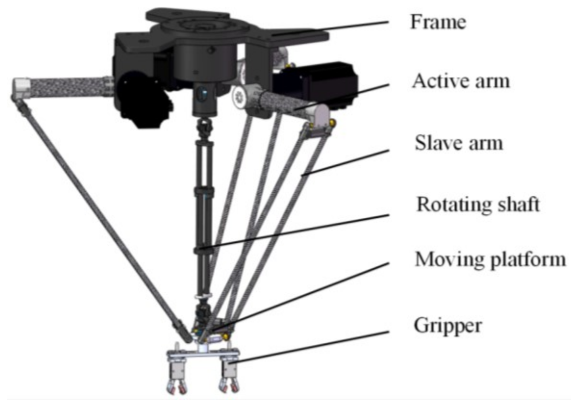


Figure 1: Three dimensional model of Delta parallel robot

The mechanism model diagram of Delta parallel robot is shown in Figure 2. The static platform takes the midpoint O as the coordinate origin, with the X-axis passing through the point E_1 and the Z-axis perpendicular to the static platform, forming a coordinate system. Similarly, the coordinate of the center O 'of the moving platform is $(x_{o'}, y_{o'}, z_{o'})$, the X'-axis of its moving coordinate system passes through a point G_1 , and the Z'-axis is perpendicular to the moving platform. At the same time, because the static platform can only move horizontally, the surfaces of the static and moving platforms are parallel, the Z-axis is parallel to the Z '-axis in space, and the establishment of the two coordinate systems follow both the right-hand rule. The distance between the coordinate origin O of the static platform and the center E_i of the rotating pair of the active arm is $OE_i = R$, the distance between the coordinate origin O' of the moving platform and the center G_i of a pair of spherical pairs of the slave arm is $O'G_i = r$, the length of active arm is $l_a = E_iF_i$, the length of slave arm is $l_b = F_iG_i$, and the angle between the three active arms and the static platform are denoted as β_i , the $Rot(z,\alpha_i)=\begin{bmatrix}\cos\alpha_i & -\sin\alpha_i & 0\\ -\sin\alpha_i & \cos\alpha_i & 0\\ 0 & 0 & 1\end{bmatrix}$ is the rotation transformation matrix of the slave arm i in the moving coordinate system relative to the static coordinate system around the Z-axis, here, α_i is the angle between the slave arm i and the Z-axis, $i=1,2,3$.

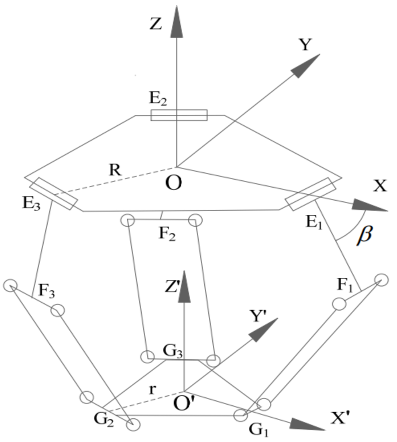


Figure 2: Schematic diagram of Delta parallel robot mechanism

Next, we will use the method of analytical mechanics to derive the dynamic equations of the Delta robotic arm. The non-independent augmented generalized coordinates of the system are taken as $\mathbf{q}=[x_{o'},y_{o'},z_{o'},\beta_1,\beta_2,\beta_3]^T$, and if the potential energy of the plane where the static platform is located is 0, then the kinetic energy T, potential energy U, and Lagrange function L of the system are respectively:

$$\begin{aligned}
&T=\frac{1}{2}m_{o'}(\dot{x}_{o'}{}^2+\dot{y}_{o'}{}^2+\dot{z}_{o'}{}^2)\\
&+\sum_{i=1}^{3}[\frac{1}{2}J_a\dot{\beta}_i^2+\frac{1}{2}m_b v_{bi}^2+\frac{1}{2}J_b\omega_{bi}{}^2]\\
&=\frac{1}{2}m_{o'}(\dot{x}_{o'}{}^2+\dot{y}_{o'}{}^2+\dot{z}_{o'}{}^2)\\
&+\sum_{i=1}^{3}[\frac{1}{2}\cdot\frac{1}{3}m_a l_a^2\dot{\beta}_i^2+\frac{1}{6}m_b(\mathbf{v}_{ui}^2+\mathbf{v}_{di}^2+\mathbf{v}_{ui}^T\mathbf{v}_{di})],\\
&\mathbf{v}_{di}=-l_a\beta_i\begin{bmatrix}\sin\beta_i\\ 0\\ \cos\beta_i\end{bmatrix},\mathbf{v}_{di}=Rot(z,\alpha_i)\begin{bmatrix}\dot{x}_{o'}\\ \dot{y}_{o'}\\ \dot{z}_{o'}\end{bmatrix},\\
&U=m_{o'}gz_{o'}+\sum_{i=1}^{3}[-\frac{1}{2}m_a g l_a\sin\beta_i+\frac{1}{2}m_b(z_{o'}-l_a\sin\beta_i)],\\
&L=T-U.
\end{aligned}\tag{1}$$

Here, $m_{o'},m_a,m_b$ is the mass of the moving platform, the active arm, and the slave arm respectively, g is the gravitational acceleration.

For any driven motion or forced vibration system, according to the second type of Lagrange equations in analytical mechanics [17]:

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{q}_j}-\frac{\partial L}{\partial q_j}=Q_j.\tag{2}$$

Generalized external forces of the system are:

$$\begin{aligned}
&Q_1=Q_2=Q_3=0,\\
&Q_4=M_4,Q_5=M_5,Q_6=M_6.
\end{aligned}\tag{3}$$

Here, M is the rotational driving torque.

It should be emphasized that the Delta robotic arm has obvious geometric constraints:

$$\begin{aligned}
&\overrightarrow{OE_i}+\overrightarrow{E_iF_i}+\overrightarrow{F_iG_i}+\overrightarrow{G_iO'}=\overrightarrow{OO'}\\
&\Rightarrow f_i(\mathbf{q})=(x_{o'}-b_{ix})^2+(y_{o'}-b_{iy})^2\\
&+(z_{o'}-b_{iz})^2-l_b^2=0\\
&b_{ix}=(R+l_a\cos\beta_i-r)\cos\alpha_i,\\
&b_{iy}=(R+l_a\cos\beta_i-r)\sin\alpha_i,\\
&b_{iz}=-l_a\sin\beta_i.
\end{aligned}\tag{4}$$

Substituting the equations (1), (3), and (4) into the equations (2), the standard form of Delta Delta parallel robot dynamics can be obtained as:

$$\mathbf{W}(\mathbf{q})\begin{bmatrix}\ddot{\beta}_1\\ \ddot{\beta}_2\\ \ddot{\beta}_3\end{bmatrix}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\begin{bmatrix}\dot{\beta}_1\\ \dot{\beta}_2\\ \dot{\beta}_3\end{bmatrix}+\mathbf{K}(\mathbf{q})=\begin{bmatrix}M_4\\ M_5\\ M_6\end{bmatrix},$$

$$\mathbf{W}(\mathbf{q})=\frac{1}{3}(m_a+m_b)l_a^2\mathbf{E}+\frac{1}{6}m_b l_a\mathbf{JP}$$

$$+(m_{o'}+m_b)(\mathbf{JJ}^T)^{-1}+\frac{1}{6}m_b l_a\mathbf{P}^T\mathbf{J}^{-1},$$

$$\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})=(m_{o'}+m_b)\mathbf{J}\dot{\mathbf{J}}^{-1}+\frac{1}{6}m_b l_a\mathbf{P}^T\dot{\mathbf{J}}^{-1}+\frac{1}{6}m_b l_a\mathbf{J}\dot{\mathbf{P}},$$

$$\mathbf{K}(\mathbf{q})=\mathbf{J}\begin{bmatrix}0\\ 0\\ m_{o'}+1.5m_b g\end{bmatrix}-\frac{1}{2}(m_a+m_b)gl_a\begin{bmatrix}\cos\beta_1\\ \cos\beta_2\\ \cos\beta_3\end{bmatrix},$$

$$\mathbf{E}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix},\mathbf{J}=\begin{bmatrix}e_1&0&0\\0&e_2&0\\0&0&e_3\end{bmatrix}\left(\begin{bmatrix}h_{11}&h_{12}&h_{13}\\h_{21}&h_{22}&h_{23}\\h_{31}&h_{32}&h_{33}\end{bmatrix}^T\right)^{-1},$$

$$h_{i1}=\left[x_{o'}+(r-R)\cos\alpha_i\right]-l_a\cos\alpha_i\cos\beta_i,$$

$$h_{i2}=\left[y_{o'}+(r-R)\sin\alpha_i\right]-l_a\sin\alpha_i\sin\beta_i,$$

$$h_{i3}=z_{o'}+l_a\sin\beta_i,$$

$$e_i=h_{i1}l_a\cos\alpha_i\sin\beta_i+h_{i2}l_a\sin\alpha_i\sin\beta_i+h_{i3}l_a\cos\beta_i,$$

$$\mathbf{P}=\begin{bmatrix}-\cos\alpha_1\sin\beta_1 & -\cos\alpha_2\sin\beta_2 & -\cos\alpha_3\sin\beta_3\\ \sin\alpha_1\cos\beta_1 & \sin\alpha_2\cos\beta_2 & \sin\alpha_3\cos\beta_3\\ \cos\beta_1 & \cos\beta_2 & \cos\beta_3\end{bmatrix}. \quad (5)$$

Here, $\mathbf{W}(\mathbf{q})$ is the inertia matrix, $\mathbf{W}(\mathbf{q})\ddot{\mathbf{q}}$ is the inertial forces proportional to the generalized acceleration, $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ is the Coriolis matrix, $\mathbf{C}(\mathbf{q})\dot{\mathbf{q}}$ is the centrifugal forces and Coriolis forces, and $\mathbf{K}(\mathbf{q})$ is the gravity matrix.

By combining the equations (4) and (5), we obtain a second-order differential algebraic equations for the dynamic driving motion of Delta parallel robot. It is a typical multi-body system dynamics equation in the form of index-3, and its numerical solution is very meaningful.

3.2 Lie Symmetries and Conserved Quantities of the System

For the equations (5), taking a infinitesimal transformation about time and generalized coordinates:

$$\begin{aligned}t^* &= t+\Delta t = t+\varepsilon\xi_0(t,\mathbf{q},\dot{\mathbf{q}}), q_s^*\\ &= q_s(t)+\Delta q_s = q_s(t)+\varepsilon\xi_s(t,\mathbf{q},\dot{\mathbf{q}}), s=1,2,3.\end{aligned} \quad (6)$$

Here, ε is an infinitesimal parameter, ξ_0,ξ_s are the generators of infinitesimal transformation.

Now, Using the extensive theory of Lie group, extending the generator of equation (6) to a second-order vector field:

$$\begin{aligned}X &= \xi_0(t,\mathbf{q},\dot{\mathbf{q}})\frac{\partial}{\partial t}+\xi_s(t,\mathbf{q},\dot{\mathbf{q}})\frac{\partial}{\partial q_s},\\ X^{(1)} &= X+(\dot{\xi}_s-\dot{q}_s\dot{\xi}_0)\frac{\partial}{\partial \dot{q}_s},\\ X^{(2)} &= X^{(1)}+(\ddot{\xi}_s-2\ddot{q}_s\dot{\xi}_0-\dot{q}_s\ddot{\xi}_0)\frac{\partial}{\partial \ddot{q}_s}.\end{aligned} \quad (7)$$

The Lie symmetry of Delta parallel robot dynamics refers to the invariance of the equations (5) and constraint conditions (4) under the transformation (6), namely:

$$\begin{aligned}&X[f_i(\mathbf{q})]=0,\\ &X^{(2)}\left[\mathbf{W}(\mathbf{q})\begin{bmatrix}\ddot{\beta}_1\\ \ddot{\beta}_2\\ \ddot{\beta}_3\end{bmatrix}2+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\begin{bmatrix}\dot{\beta}_1\\ \dot{\beta}_2\\ \dot{\beta}_3\end{bmatrix}+\mathbf{K}(\mathbf{q})-\begin{bmatrix}M_4\\ M_5\\ M_6\end{bmatrix}\right]=0.\end{aligned} \quad (8)$$

The equations (8) are a set of Lie symmetry determining equations of Delta parallel robot dynamics for the generator functions ξ_0,ξ_1,ξ_2,ξ_3 under keeping constraints unchanged.

Furthermore, based on the twin relationship between symmetry and conserved quantity, the conditions and the forms of Noether type conserved quantity corresponding to Lie symmetry can be given.

Theorem 1: If the generator functions ξ_0,ξ_s of infinitesimal transformation satisfy the Lie symmetry determining equations (8) of the Delta parallel robot system, and there exists a gauge function that satisfies the following structural equation, namely:

$$L\dot{\xi}_0+X^{(1)}(L)+(\frac{d\partial L}{dt\partial\dot{q}_s}-\frac{\partial L}{\partial q_s})(\xi_s-\dot{q}_s\xi_0)+\dot{V}=0. \quad (9)$$

Then, corresponding to Lie symmetry, the Delta parallel robot dynamics equations (5) have a conserved quantity in the following form:

$$I_N=L\xi_0+(\frac{\partial L}{\partial\dot{q}_s})(\xi_s-\dot{q}_s\xi_0)+V=const. \quad (10)$$

Proof: Taking the total derivative of equation (10), we have:

$$\begin{aligned}\frac{dI_N}{dt} &= \dot{L}\xi_0+L\dot{\xi}_0+\frac{d}{dt}[(\frac{\partial L}{\partial\dot{q}_s})(\xi_s-\dot{q}_s\xi_0)]+\dot{V}\\ &= \dot{L}\xi_0+L\dot{\xi}_0+(\frac{d\partial L}{dt\partial\dot{q}_s})(\xi_s--\dot{q}_s\xi_0)\\ &+(\frac{\partial L}{\partial\dot{q}_s})(\dot{\xi}_s-\ddot{q}_s\xi_0-\dot{q}_s\dot{\xi}_0)+\dot{V}.\end{aligned}$$

Using equation (9), it can be obtained that:

$$\begin{aligned}\frac{dI_N}{dt} &= \dot{L}\xi_0+L\dot{\xi}_0+(\frac{d\partial L}{dt\partial\dot{q}_s})(\xi_s-\dot{q}_s\xi_0)\\ &+(\frac{\partial L}{\partial\dot{q}_s})(\dot{\xi}_s-\ddot{q}_s\xi_0-\dot{q}_s\dot{\xi}_0)\\ &-L\dot{\xi}_0-\frac{\partial L}{\partial t}\xi_0-\frac{\partial L}{\partial q_s}\xi_s-\frac{\partial L}{\partial\dot{q}_s}(\dot{\xi}_s-\dot{q}_s\dot{\xi}_0)\\ &-(\frac{d\partial L}{dt\partial\dot{q}_s}-\frac{\partial L}{\partial q_s})(\xi_s-\dot{q}_s\xi_0)\\ &= \dot{L}\xi_0-\frac{\partial L}{\partial t}\xi_0-(\frac{\partial L}{\partial q_s})\dot{q}_s\xi_0-(\frac{\partial L}{\partial\dot{q}_s})\ddot{q}_s\xi_0\\ &= [\dot{L}-\frac{\partial L}{\partial t}-(\frac{\partial L}{\partial q_s})\dot{q}_s-(\frac{\partial L}{\partial\dot{q}_s})\ddot{q}_s]\xi_0=0.\end{aligned}$$

3.3 The Conserved-Quantity-Preserving Algorithms

Firstly, introducing the generalized momentum $\mathbf{p}=\frac{\partial L}{\partial\dot{\mathbf{q}}}=\mathbf{W}(\mathbf{q})\dot{\mathbf{q}}$, and using the Legendre transformation, the Hamiltonian function of Delta parallel robot dynamics is:

$$H(t,\mathbf{q},\mathbf{p}) = \dot{q}_s(t,\mathbf{q},\mathbf{p})p_s - L \tag{11}$$

According to the Hamilton principle, the canonical equations form of the constrained Delta parallel robot dynamics system is:

$$\begin{aligned}
&\dot{\mathbf{q}} = \frac{\partial H}{\partial \mathbf{p}} = \mathbf{W}(\mathbf{q})^{-1}\mathbf{p},\\
&\dot{\mathbf{p}} = -\frac{\partial H}{\partial \mathbf{q}} + \mathbf{Q} = \dot{\mathbf{W}}(\mathbf{q})\mathbf{W}(\mathbf{q})^{-1}\mathbf{p} - \mathbf{C}(\mathbf{q},\mathbf{p})\mathbf{q} - \mathbf{K}(\mathbf{q}) + \mathbf{Q},\\
&\mathbf{f}(\mathbf{q}) = 0.
\end{aligned} \tag{12}$$

By substituting the generalized momentum and Hamiltonian function of the system into the conserved quantity equation (10), the conserved quantity can be transformed into:

$$I_N = p_s\xi_s - H\xi_0 + V = const. \tag{13}$$

For the equations (12), we consider a conserved-quantity-preserving numerical algorithm for equations (13) by coordinate increment discrete gradient as follows [18-20]:

$$\begin{aligned}
&\mathbf{q}_{n+1} = \mathbf{q}_n n + \Delta t \mathbf{W}(\mathbf{q}_n)^{-1}\mathbf{p}_{n+1/2},\\
&\mathbf{p}_{n+1/2} = \mathbf{p}_n - \frac{\Delta t}{2}\bar{\nabla} I_N(t_n,\mathbf{p}_n,\frac{\mathbf{q}_{n+1}+\mathbf{q}_n}{2})\\
&-\frac{\Delta t}{2}\mathbf{f}_q^T(t_n,\frac{\mathbf{q}_{n+1}+\mathbf{q}_n}{2})\boldsymbol{\lambda},\\
&\mathbf{p}_{n+1} = \mathbf{p}_{n+1/2} - \frac{\Delta t}{2}\sigma\bar{\nabla} I_N(t_n,\mathbf{p}_n,\frac{\mathbf{q}_{n+1}+\mathbf{q}_n}{2})\\
&-\frac{\Delta t}{2}\sigma\mathbf{f}_q^T(t_n,\frac{\mathbf{q}_{n+1}+\mathbf{q}_n}{2})\boldsymbol{\lambda},\\
&\mathbf{f}(\mathbf{q}_{n+1}) = 0, n = 0,1,2\ldots,\\
&\Delta t = t_{n+1} - t_n,\\
&\sigma = \frac{2\Delta t(U(\mathbf{q}_n) - U(\mathbf{q}_{n+1}))}{(\mathbf{q}_{n+1} - \mathbf{q}_n - \Delta t\mathbf{W}(\mathbf{q}_n)^{-1}\mathbf{p}_n)^T}(\mathbf{p}_{n+1}+\mathbf{p}_n) - 1,\\
&\bar{\nabla} I_N(t_n,\mathbf{q}_{n+1},\mathbf{p}_{n+1}) = \begin{bmatrix} \dfrac{I_N(t_n,\mathbf{q}_{n+1},\mathbf{p}_{n+1}) - I_N(t_n,\mathbf{q}_n,\mathbf{p}_{n+1})}{\mathbf{q}_{n+1}-\mathbf{q}_n} \\ \dfrac{I_N(t_n,\mathbf{q}_{n+1},\mathbf{p}_{n+1}) - I_N(t_n,\mathbf{q}_{n+1},\mathbf{p}_n)}{\mathbf{p}_{n+1}-\mathbf{p}_n} \end{bmatrix},\\
&\mathbf{f}_{\mathbf{q}} = \begin{bmatrix} \partial f_1/\partial q_1 & \partial f_1/\partial q_2 & \partial f_1/\partial q_3 \\ \partial f_2/\partial q_1 & \partial f_2/\partial q_2 & \partial f_2/\partial q_3 \\ \partial f_3/\partial q_1 & \partial f_3/\partial q_2 & \partial f_3/\partial q_3 \end{bmatrix}.
\end{aligned} \tag{14}$$

Here, Δt is the step size, $\boldsymbol{\lambda}$ is the Lagrange multiplier. The equations (14) contain a total of nine nonlinear algebraic equations, of which $\mathbf{p}_n, \mathbf{q}_n, \boldsymbol{\lambda}$ are the variables. The above nonlinear algebraic equations can be solved by using the Newton-Raphson iterative method.

Furthermore, we derive that the iterative format in equations (14) is conserved-quantity-preserving. By eliminating $\mathbf{p}_{n+1/2}$ twice in equations (14), we can obtain:

$$\mathbf{p}_{n+1} - \mathbf{p}_n = (\sigma+1)\frac{\mathbf{W}(\mathbf{q}_n)(\mathbf{q}_{n+1} - \mathbf{q}_n - \Delta t\mathbf{W}(\mathbf{q}_n)^{-1}\mathbf{p}_n)}{\Delta t} \tag{15}$$

Furthermore, taking a class of special generator functions and gauge function in Lie symmetry determining equations and structural equation $\xi_0 = -1, \xi_1 = \xi_2 = \xi_3 = 0, V = 0$, so the equations (13) for the conserved quantity caused by Lie symmetry can be expressed as:

$$I_N(t,\mathbf{q},\mathbf{p}) = H(t,\mathbf{q},\mathbf{p}) \tag{16}$$

So, the discrete iterative of conserved quantity can be expressed as:

$$\begin{aligned}
&I_N(t_{n+1},\mathbf{q}_{n+1},\mathbf{p}_{n+1}) - I_N(t_n,\mathbf{q}_n,\mathbf{p}_n)\\
&= H_{n+1}(t_{n+1},\mathbf{q}_{n+1},\mathbf{p}_{n+1}) - H_n(t_n,\mathbf{q}_n,\mathbf{p}_n)\\
&= \frac{1}{2}(\mathbf{p}_{n+1}-\mathbf{p}_n)^T\mathbf{W}(\mathbf{q}_n)^{-1}(\mathbf{p}_{n+1}+\mathbf{p}_n)^T + U(\mathbf{q}_{n+1}) - U(\mathbf{q}_{n+1})\\
&= U(\mathbf{q}_{n+1}) - U(\mathbf{q}_n)\\
&+\frac{1}{2}\left[\frac{\dfrac{2\Delta t(U(\mathbf{q}_n) - U(\mathbf{q}_{n+1}))}{(\mathbf{q}_{n+1}-\mathbf{q}_n-\Delta t\mathbf{W}(\mathbf{q}_n)^{-1}\mathbf{p}_n)^T(\mathbf{p}_n+\mathbf{p}_{n+1})}\mathbf{W}(\mathbf{q}_n)(\mathbf{q}_{n+1}-\mathbf{q}_n-\Delta t\mathbf{W}(\mathbf{q}_n)^{-1}\mathbf{p}_n)^T}{\Delta t}\right]\\
&\cdot\mathbf{W}(\mathbf{q}_n)^{-1}(\mathbf{p}_n+\mathbf{p}_{n+1}) = U(\mathbf{q}_{n+1}) - U(\mathbf{q}_n)\\
&+\frac{(U(\mathbf{q}_n) - U(\mathbf{q}_{n+1}))}{(\mathbf{p}_n+\mathbf{p}_{n+1})}\mathbf{W}^T(\mathbf{q}_n)\mathbf{W}(\mathbf{q}_n)^{-1}(\mathbf{p}_n+\mathbf{p}_{n+1})\\
&= U(\mathbf{q}_{n+1}) - U(\mathbf{q}_n) + U(\mathbf{q}_n) - U(\mathbf{q}_{n+1}) = 0
\end{aligned} \tag{17}$$

Therefore, the numerical format (14) can keep the system's conserved quantity unchanged.

4. Example Simulation

The material of a industrial 3T Delta parallel manipulator for grippering is 45 # steel, and its shear modulus is 80Gpa. Other main geometric and physical parameters are shown in Table 1.

Table 1. The dynamic parameters of Delta parallel robot

Moving platform mass /kg	Active arm length /m	Slave arm length /m	Active arm mass /kg
$m_{o'} = 1$	$l_a = 0.3$	$l_b = 0.9$	$m_a = 1.2$
Slave arm mass /kg	Moving platform radius /m	Static platform radius /m	Angles around Z-axis /rad
$m_b = 0.2$	$r = 0.05$	$R = 0.3$	$\alpha_1 = \pi/6$, $\alpha_2 = 5\pi/6$, $\alpha_3 = -\pi/2$

The initial full state vector of the system dynamic response is $\mathbf{q}^{(0)} = [\pi/3, \pi/6, \pi/4,]^T$, the gravity acceleration is $g = 10m/s^2$, the step size Δt is 0.01s, the external driving torque function is

$M_4 = 2\sin(2\pi t + \pi/6) + 7,$

$M_5 = 2\sin(2\pi t + 5\pi/6) + 5,$, and the total number of

$M_6 = 2\sin(2\pi t - \pi/2) + 7$

steps is 200. According to the conserved-quantity-preserving algorithm in equations (14), the time history curve of Delta parallel mechanical manual mechanical response is simulated in Matlab software, as shown in Figure 3.

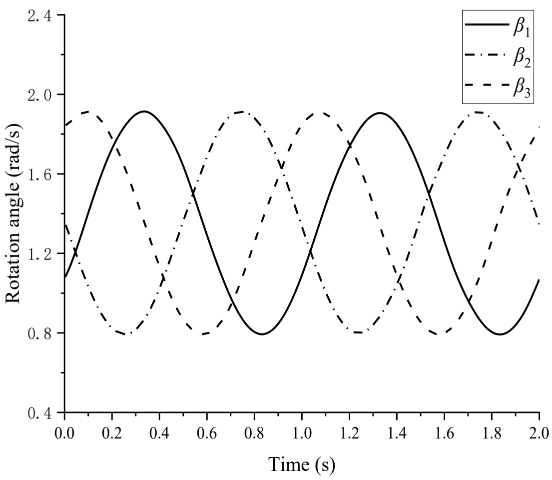
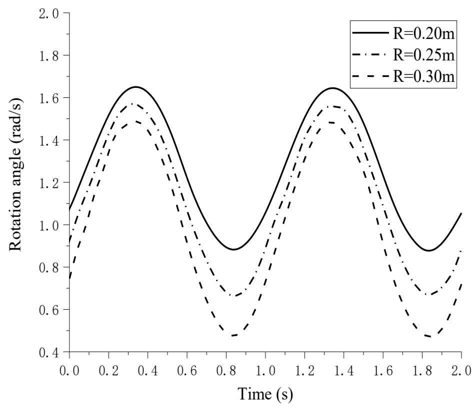


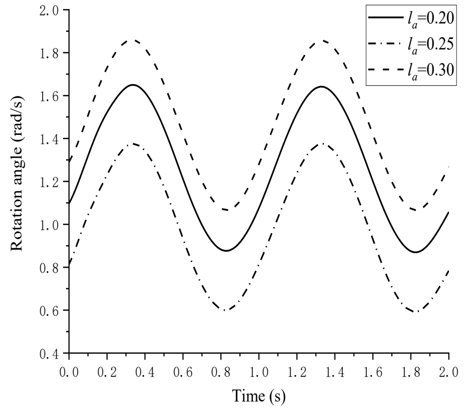
Figure 3: Joint rotation angle response Curve of Delta parallel robot

From Figure 3, it can be seen that the joint rotation angles β_i of the three active arms all vary in a sinusoidal pattern with a period of 2π, and their phases are $2\pi/3$ behind each other in sequence, which is consistent with the 120 ° symmetrical distribution of the three driving joints on the static platform. Further, utilizing the geometric constraint equations (4), we can obtain $\begin{cases} x_{o'} = 0.1\cos 2\pi t \\ y_{o'} = 0.1\sin 2\pi t \\ z_{o'} = -0.7 \end{cases}$, so the moving platform performs a circular motion with a radius of 0.1m in the horizontal plane, thus this Delta robotic arm can achieve the translational motion.

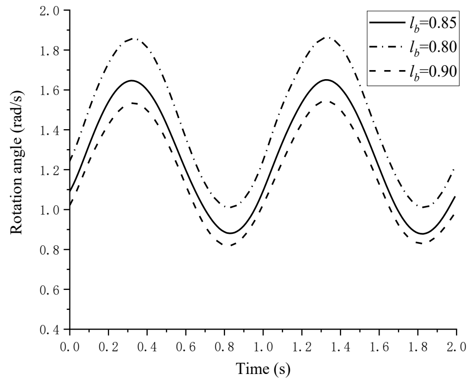
While keeping other dynamic parameters constant, sequentially change the static platform radius, active arm length, slave arm length and moving platform radius of the Delta robotic arm. The variation curve β_1 by using the conserved-quantity-preserving numerical iteration algorithm according to the equations (14) is shown in Figure 4.



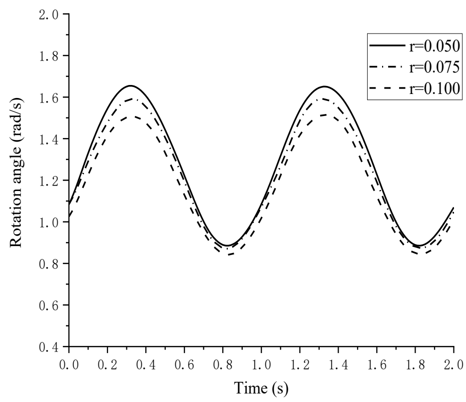
(a) Static platform radius



(b) Active arm length



(c) Slave arm length

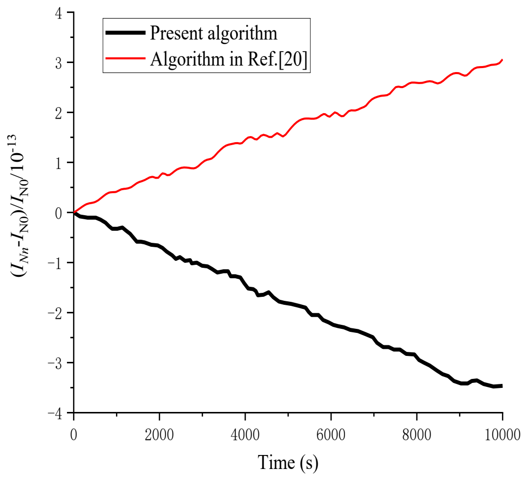


(d) Moving platform radius

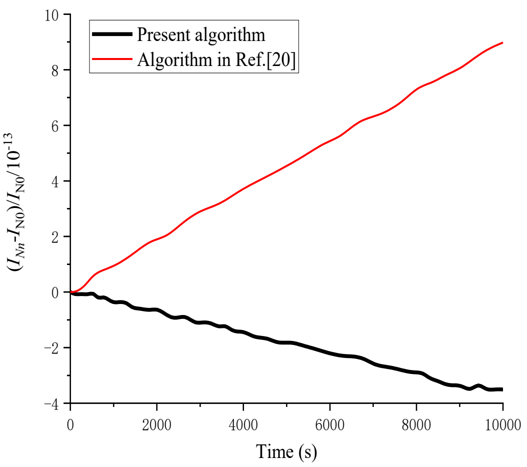
Figure 4: The variation curve of joint rotation angle β_1 at different levels of dynamic parameters

From Figure 4, it can be seen that the numerical value of the driving joint rotation angle is negatively correlated with the static platform radius, the slave arm length, and the moving platform radius, but positively correlated with the active arm length. Among these four parameters, the length of the active arm length has a relatively maximum impact on the rotation angle, while the moving platform radius has a relatively minimal impact. These analysis results have guiding significance for the optimization design of robot. In order to reduce the output torque of the motor while meeting the conditions of workspace, flexibility, and non-singular, relatively larger sizes should be selected for the static platform radius and the moving platform radius, as well as the slave arm length, while smaller sizes should be selected for the active arm length.

Considering the long-term behavior of the algorithm, step size Δt is used 0.01s and 0.1s spectively. When the running time is 100s, the relative error of system's conserved quantity $I_N(\mathbf{q},\mathbf{p})$ iteration is shown in Figure 5.



(a) $\Delta t = 0.01s$



(b) $\Delta t = 0.1s$

Figure 5: Relative iteration error of conserved quantity

From Figure 5, it can be seen that the error of the algorithm proposed in this paper is a certain decrease compared to the algorithm proposed in reference [20], and the error tends of conserved-quantity-preserving to converge. The conserved-quantity-preserving numerical algorithm constructed in this paper maintains a relative error of 10-13 order of magnitude for the conserved quantity of system dynamics during iteration process. Meanwhile, with different step sizes, the conserved quantity iteration format is not sensitive to step size response, so this algorithm is also suitable for large step size calculations and can effectively improve computational efficiency. In summary, the high-precision numerical method of conserved-quantity-preserving for Lie symmetry constructed by using discrete gradients does not introduce artificial dissipation over time and has good long-term tracking ability.

5. Conclusions

This article applies the Lie symmetry analysis method to study the nonlinear dynamic characteristics of the 3T type of Delta robot, obtains the conserved quantities of the system under closed chain constraints of motion, and constructs a structure-preserving numerical algorithm based on the conserved quantity. The main conclusions are as follows:

(1) The Lagrange equations for the rigid body dynamics of Delta robot system are the second-order nonlinear coupled ordinary differential algebraic equations, which is consistent with the form of the multi-body system dynamics index-3 problem.

(2) The Lie symmetry criterion of the system is given through the invariance of differential and algebraic equations. The equations have some open solutions, and the corresponding Noether type conserved quantity is a first integral of the dynamic response, which is also an extension of the energy conservation law of the conservative system of Delta robot.

(3) Numerical simulation shows that under the constraints conditions, the direction of optimizing the dynamic characteristics of Delta parallel robot is to increase the radius of the static and moving platforms, as well as the length of the slave arm, while reducing the size of the active arm.

(4) The conserved-quantity-preserving numerical solutions of dynamic equations based on discrete gradient can simultaneously preserve constraints and conservative constants, and it can be stable for different step sizes. The system energy deviation does not diffuse, and the long-term simulation process of the system is not distorted.

It should be noted that the method of Lie symmetry and conserved quantity is easy to standardize, and with the introduction of symbolic software technology, solving the above process will be highly efficient. We can not only obtain the first integrals of the system's dynamic response, but also lay the foundation for efficient and accurate numerical simulation calculations. Furthermore, the research method proposed in this article can be extended to the calculation of robot dynamics under complex geometric and motion constraints (non-ideal).

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