International Journal of Mechatronics and Applied Mechanics
2025, Issue 22, Vol. I

METHODOLOGY FOR IDENTIFYING AND ELIMINATING BRAKE SQUEAL IN WHEELED VEHICLES

Viktor Bogomolov[1][0000-0001-6871-8244], *Dmytro Leontiev*[1] [0000-0003-4255-6317], *Oleksandr Yaryta*[1] [0000-0003-4948-6577], *Aleksandra Kuzior*[1,2][0000-0001-9764-5320], *Oleksandr Pysartsov*[1][0000-0003-4661-5441], *Yevhen Don*[1][0000-0002-1508-8397]
[1]*Kharkiv National Automobile and Highway University, Ukraine*
[2]*Silesian University of Technology, Republic of Poland*
Email: leontiev@khadi.kharkov.ua

Abstract This paper examines the influence of design and operational parameters of drum brake mechanisms on dynamic stability and the occurrence of brake squeal. The focus is on the leading (primary) shoe, which is a critical component determining the level and stability of braking torque. An analysis was conducted of the dependence of braking torque on the shoe installation angle β and geometric parameters, taking into account the stochastic distribution of the friction coefficient μ and the effective friction radius ρ_y, allowing for more accurate modeling of real operating conditions of the brake mechanism. Experimental studies performed on a heavy-duty truck confirmed that at small angles β < 10 deg, self-locking and brake squeal occur, whereas a range of 15 – 25 deg, ensures a stable distribution of contact stresses and eliminates undesirable self-excited vibrations. For the trailing shoe, the probability of self-locking is minimal, which allows for simplified engineering calculations, although its influence on braking dynamics under high loads should still be considered. The proposed methodology can be extended using numerical modeling and machine learning to analyze large datasets, account for shoe wear, nonlinear effects, and changes in material properties over time. This approach enables the prediction of not only short-term self-excited vibrations but also the steady-state development of braking torque, enhancing the reliability and durability of brake systems for heavy-duty vehicles.

Keywords: Drum brake, Brake squeal, Leading shoe, Trailing shoe, Friction coefficient, Dynamic stability, Heavy-duty vehicles.

1. Introduction

When a vehicle is braking, a so-called squeal of its braking mechanisms sometimes occurs [1], accompanied by undesirable and harmful vibrations of their components [2]. This process represents a complex acoustical-mechanical phenomenon associated with the interaction of the pads and brake discs or drums, when oscillations appear in the system that go beyond the permissible range. Such sound effects are observed both during partial and intensive braking of the vehicle, and their occurrence largely depends on the brake design, the condition of its friction linings, the quality of brake component manufacturing, as well as the vehicle's operating conditions. The resulting noise not only causes discomfort to the driver and passengers during vehicle operation [3], but also has a negative impact on the overall assessment of the vehicle. For the modern consumer, not only the reliability and safety of a vehicle are important, but also its comfort, which includes acoustic characteristics. Therefore, even in the absence of an immediate threat to road safety, the presence of extraneous noises is perceived as a serious drawback. This is especially noticeable under urban driving conditions, where braking occurs very frequently, and the driver is forced to regularly deal with unpleasant sound effects accompanying the operation of the braking system.

From the point of view of the vehicle manufacturer, this factor is also highly undesirable [4, 9]. Firstly, the presence of brake squeal creates a negative brand image, reducing the level of customer trust and loyalty. Secondly, customers very often file complaints on this issue, demanding that the defect be eliminated [1, 4]. Such claims lead to the need for additional service work, warranty replacements of components, or even modernization of assemblies, which entails direct material costs. As a result, the manufacturer suffers losses associated not only with additional customer service expenses but also with

https://doi.org/10.17683/ijomam/issue22.v1.17

indirect losses caused by a decline in brand reputation.

It is obvious that in such cases the manufacturer incurs significant non-productive costs [7, 9]. These are expressed in the increased cost of after-sales service, a rise in the number of warranty claims, the need to revise design solutions, and adjustments to technological processes. All this together leads to reduced profitability of production and decreased competitiveness of the enterprise in the market.

In the context of the modern transition to the concept of sustainable development, it is especially important to take into account not only the economic but also the environmental aspect of the problem under consideration. Excessive noise generated during brake operation contributes to overall environmental noise pollution, which is recognized as one of the significant factors negatively affecting human health and quality of life. Eliminating this phenomenon not only improves the comfort of vehicle operation but also reduces the negative impact of noise on the urban environment.

Moreover, sustainable development presupposes the rational use of resources and the reduction of non-productive costs. Any additional expenses for the maintenance and refinement of braking systems decrease the economic efficiency of production and lead to the overuse of materials, energy, and labor resources. Consequently, reducing the likelihood of brake squeal meets the goals of sustainable development, as it allows for the simultaneous improvement of the social, environmental, and economic indicators of the industry.

Thus, the study of the causes of brake squeal and the development of methods to prevent it is not only an engineering task aimed at improving the quality and reliability of vehicles but also an important component of sustainable development policy, which is focused on the harmonious balance of the interests of consumers, manufacturers, and society as a whole.

2. Analysis of Publications

The so-called brake squeal (noise, screech, vibration), which may arise during vehicle braking [5-9], is accompanied by the occurrence of intensive vibrations in the brake mechanism components [5, 8, 10-12]. This, in turn, accelerates wear of the elements and reduces their operational durability [2, 13]. In this regard, a well-founded position has been put forward in the scientific literature that the probability of brake squeal occurrence should be taken into account already at the stage of system-level brake design [4, 6, 9, 15, 16].

The key factor in its emergence, as noted by most researchers, is the nonlinearity and instability of friction processes [17] occurring in braking mechanisms [1, 3, 4, 8, 11]. It has been established that the instability of the braking torque [1, 6, 7, 11, 15, 18, 20, 29] directly affects the dynamic stability of the vehicle during braking [19] and, consequently, the overall road traffic safety [2, 7, 15, 16, 30].

This circumstance determines the long-standing and continuing scientific interest in the problem of brake squeal, as confirmed by numerous theoretical and experimental studies [1, 3-8, 10-12, 14, 16, 18].

Among the factors contributing to the generation and development of vibration processes, in addition to the instability of friction pairs, the following are distinguished:

- contact interaction conditions (normal pressure and sliding velocity of surfaces) [1, 6, 11, 30];
- stiffness and resonance characteristics of brake mechanism components [3-5, 9-11, 14, 19];
- physical and mechanical properties of brake element materials [4, 8, 9, 28];
- structural and geometrical parameters of brake assemblies [4, 9, 16, 27];
- surface conditions of friction interfaces, including microgeometry and roughness [11, 13, 21, 25];
- effective area of friction linings [14, 15, 17].

At the same time, as noted in a number of publications [2-4], the root causes of brake squeal have not yet been fully determined.

It should also be emphasized that the majority of studies have been primarily focused on disc brake systems, while drum brakes [20, 24, 26] remain relatively underexplored. This fact is of particular importance considering the wide application of drum brakes in medium- and heavy-duty trucks [21, 22, 29], buses, and road trains [23], whose reliable operation is critical for ensuring road traffic safety.

Therefore, in the present study, the primary attention is focused on the analysis and assessment of drum brake mechanisms.

3. Objective and Problem Statement

The objective of this study is to develop a set of measures aimed at eliminating brake squeal in vehicles, using drum brake mechanisms as an example. Brake squeal is a complex acoustical-mechanical phenomenon that not only reduces driving comfort but also negatively affects consumer perception of vehicle quality. Solving this problem requires a comprehensive approach that takes into account design parameters, the properties of friction lining materials, manufacturing quality, and the operating conditions of the braking system. Ultimately, the elimination of brake squeal in vehicle braking mechanisms will contribute to improving vehicle reliability, enhancing acoustic comfort, and adhering to the principles of sustainable development in the automotive industry.

To achieve the stated objective, it is necessary to accomplish the following tasks:

- to develop a static functional model of the brake;
- to define the operating range with the most stable output parameters of the floating shoe of the drum brake;
- to examine the self-locking effect of the leading shoe of the drum brake.

4. A Static Functional Model of the Brake

In modern automobiles, two main types of drum brake mechanisms (Fig. 1) are the most widely used due to their reliability, simple design, and relatively low cost:

– Drum brake with one leading (primary) and one trailing (secondary) shoe (Fig. 1 a). This design is characterized by lower production and maintenance costs but results in an uneven distribution of loads on the shoes, causing one of them to wear out faster.

– Drum brake with two leading shoes (Fig. 1 b). These mechanisms are more often used in vehicles with higher reliability requirements for the braking system, as well as in vehicles that operate under severe road conditions. This design provides higher and more stable braking force, reduces friction lining wear, and increases braking efficiency under prolonged loads.

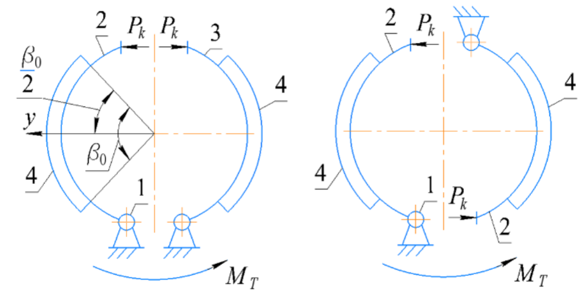


Figure 1: The most common vehicle drum brake mechanisms: a – drum brake with one leading (primary) and one trailing (secondary) shoe; b – drum brake with two leading shoes; 1 – shoe support; 2 – leading shoe; 3 – trailing shoe; 4 – brake lining; M_T – braking torque; P_k – force with which the shoe is pressed against the brake drum.

In Figure 1a, it can be seen that in a drum brake with one leading (primary) and one trailing (secondary) shoe, the direction of the force P_k of the leading shoe relative to support 1 coincides with the direction of the braking torque M_T, whereas the direction of the force P_k of the trailing shoe relative to its support is opposite to the direction of the braking torque M_T.

In the drum brake mechanisms described above, two types of brake shoes may be used:

– with one degree of freedom (as shown in Fig. 1);

– with two degrees of freedom (floating shoe) (see Fig. 2).

As a rule, a shoe that is not supported on a pivot axis (see Fig. 1) but instead has a movable flat surface (see point E in Fig. 2) is self-centering relative to the brake drum, which increases the service life of the brake linings.

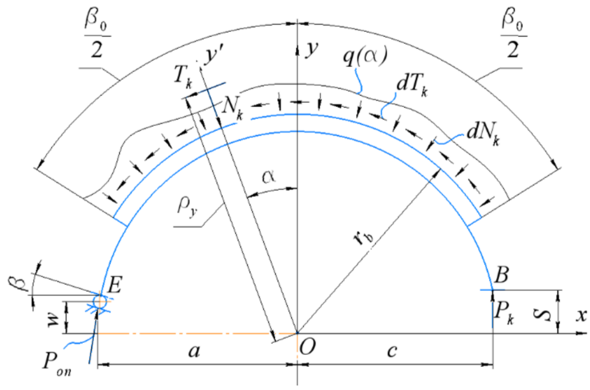


Figure 2: Model of a floating leading shoe with two degrees of freedom: $q(\alpha)$ – law of distribution of normal contact pressure on the brake lining; dN_k, $dT_k=\mu\cdot dN_k$ – elementary forces, respectively: normal and tangential (friction); N_k, T_k – resultants of the elementary forces dN_k and dT_k, respectively; ρ_y – equivalent (effective) friction radius of the shoe; β_0 – wrap angle of the brake lining; α – angle of displacement of the bisector (y) of the brake lining wrap angle.

The primary output parameter of a brake shoe is the braking torque it produces, which serves as a key indicator of the efficiency of the entire brake mechanism and has a direct impact on overall vehicle safety. As previously discussed, the phenomenon of brake squeal is closely associated with dynamic oscillations within the braking mechanism and manifests itself as instability and temporal fluctuations of this output parameter. Such oscillations not only generate the characteristic high-pitched noise but may also result in uneven braking performance, accelerated wear of friction linings, and a reduction in driver comfort and vehicle controllability.

Consequently, it is both reasonable and necessary, in order to minimize the probability of brake squeal and enhance the overall reliability of the braking mechanism, to perform a comprehensive analysis and develop engineering measures aimed at stabilizing the braking torque. These measures may involve improvements in the structural design of the brake shoes and their supports, as well as the optimization of friction material properties and the geometric characteristics of the brake drum. It has been established [16, 23] that for both major types of brake shoes used in drum brake mechanisms, the well-known equation (1) holds true, providing a theoretical basis for correlating contact force parameters, the lining wrap angle, and the resulting braking torque – an essential consideration in design calculations and performance optimization.

$$M_{k_{1,2}} = \frac{P_k\cdot(a+c)\cdot\mu\cdot\rho_y}{\cos\alpha_{1,2}\left(a\pm\mu\cdot w\right)-\sin\alpha_{1,2}\left(w\mp\mu\cdot a\right)\mp\mu\cdot\rho_y}, \quad (1)$$

where α, a, w, ρ_y – geometric parameters shown in Fig. 2; M_k – braking torque on the shoe; the subscripts 1 and 2 indicate that the parameters M_k and α correspond to the leading and trailing shoes, respectively; μ – coefficient of friction between the brake lining and the drum; ± and ∓ – the upper sign refers to the leading shoe, and the lower sign to the trailing shoe.

It is known that for a floating shoe, the following equation can be used in engineering calculations:

$$\sin\alpha_{1,2} = \frac{-F \pm \sqrt{F^2 - 4EG}}{2 \cdot E}; \tag{2}$$

where:

$$E = A^2 + B^2; \tag{3}$$

$$F = 2 \cdot A \cdot D; \tag{4}$$

$$G = D^2 - B^2; \tag{5}$$

$$A_{1,2} = c \cdot \left(1 \pm \mu \cdot \tan\beta\right) + a + w \cdot \tan\beta; \tag{6}$$

$$B_{1,2} = \pm\mu \cdot \left(w \cdot \tan\beta + a + c\right) - c \cdot \tan\beta; \tag{7}$$

$$D_{1,2} = \pm\mu \cdot \rho_y \cdot \tan\beta. \tag{8}$$

Considering M_k as a function of random variables, it can be concluded that for a shoe with one degree of freedom:

$$M_k = f(\mu, \alpha, \rho_y). \tag{9}$$

In equation (9), the parameters α and ρ_y, as well as the friction coefficient μ, are treated as random variables, primarily depending on the distribution of contact stresses in the "lining–drum" friction pair. Since this contact is discrete in nature, it exhibits stochastic behavior. Consequently, in this case, the braking torque M_k can be represented as a function of three random variables.

At the same time, for a floating shoe, taking into account equations (2) – (8), the braking torque M_k can be expressed as a function of two independent random variables:

$$M_k = f(\mu, \rho_y). \tag{10}$$

It is evident that, by design, a floating shoe exhibits more stable output characteristics and, all else being equal, is more preferable in terms of reducing the likelihood of brake squeal. However, another question arises: how should the working ranges of the shoe parameters μ and ρ_y be selected so as to ensure the most stable output characteristics of the shoe?

5. Define the Operating Range with the Most Stable Output Parameters of the Floating Shoe

The leading (primary) shoe has a significantly higher efficiency coefficient than the trailing (secondary) shoe. Moreover, it is more commonly used in drum brake designs (see Fig. 1). Thus, the leading shoe primarily determines the stability of the braking torque in a drum brake. For this reason, the subsequent analysis will focus specifically on the leading shoe.

Let us examine function (1) for extrema with respect to the argument α. It turns out that this function has a minimum when

$$\tan\alpha = \frac{\mu \cdot a - w}{\mu \cdot w + a}. \tag{11}$$

This implies that when designing a drum brake, particular attention must be paid to the geometry of the leading shoe. The shape and positioning of the shoe should be selected in such a way that the expected value of the angle α (see Fig. 2), which characterizes the direction of the resultant normal force N_k acting on the shoe, satisfies the relationship given by equation (11). Compliance with this condition ensures that the generated braking torque will remain within a stable range, thereby reducing the likelihood of self-excited vibrations and brake squeal during operation.

In other words, the criterion formulated in equation (11) establishes a fundamental design requirement, linking the geometric configuration of the brake components with the dynamic stability of the braking process. For the specific case of a leading (trailing) shoe possessing a single degree of freedom, the above consideration leads to a simplified formulation that can be expressed as follows:

– the coordinate of the bisector (y-axis) of the wrap angle β_0 (see Fig. 1a) should be shifted relative to the horizontal axis in the direction of the braking torque M_T by an amount as close as possible to the value of α calculated from equation (11);

– it is necessary to aim at reducing the magnitude of the wrap angle β_0.

For a floating leading shoe, it is necessary to aim for satisfying equation (11) at the average values of the friction coefficient μ obtained for wrap angles β (see Fig. 2) within the range from 0 to β_{max}.

Figure 3 shows an example calculation using equations (2) – (8) and (11) for a drum brake with geometric parameters typical for a heavy-duty vehicle, in particular: c = 0.15 m, r_b = 0.2 m, β_0 = 110 deg, μ = 0.35, and a wrap angle range β from 0 to 38 deg.

Figure 4 shows the recommended values of w depending on the installation angles β (see Fig. 2).

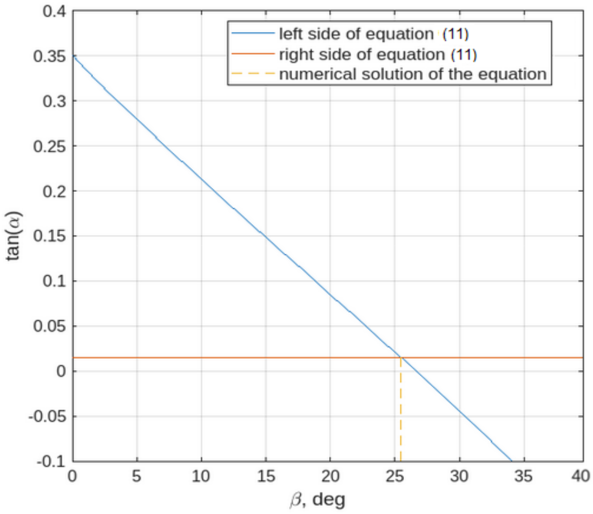


Figure 3: An example of a numerical solution of equation (11).

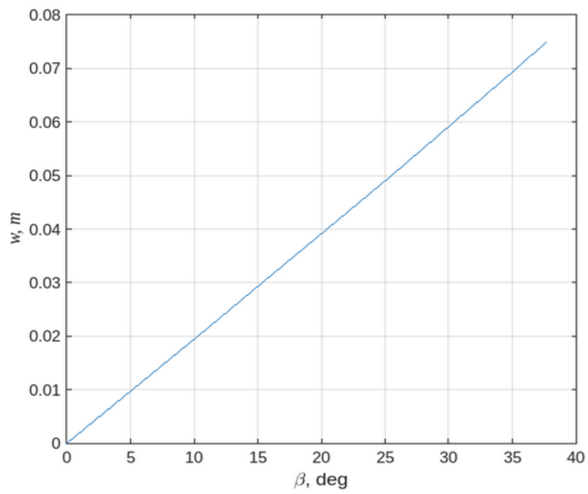


Figure 4: Recommended values of w as a function of β.

As shown in Figure 4, for angles β (the angle between the movable shoe support and the vertical axis) up to 38 deg, the value of w should not exceed 0.075 m. Maintaining this limit is important to ensure proper functioning of the leading shoe and to avoid excessive variations in the contact conditions between the brake lining and the drum. Exceeding this value could lead to uneven distribution of contact forces, reduced stability of the braking torque, and a higher likelihood of brake squeal or premature wear of the friction material.

At the same time, the optimal value of the angle α should be 26 deg (as can be seen in Fig. 3). Setting α at this value helps to maximize the efficiency of the braking system, ensuring that the resultant forces acting on the shoe are aligned in a way that provides stable and predictable torque. Deviations from this angle may reduce braking performance, affect the uniformity of shoe wear, and compromise the overall reliability of the drum brake under varying operating conditions.

6. Self-locking Effect of the Leading (primary) Shoe

In [19], it is hypothesized that the primary cause of self-excited vibrations and brake squeal may be the so-called self-locking effect of the leading shoe. In other words, under unfavorable conditions – such as a sudden change in the friction coefficient μ or an uneven redistribution of contact stresses in the "lining–brake drum" pair – a sharp jump in the braking torque M_k occurs, after which the shoe enters a self-oscillation mode. Mathematically, this can be expressed in such a way that the denominator of equation (1) for the floating leading shoe cannot theoretically become zero. To ensure this, inequality (12) must be satisfied:

$$\cos\alpha_1\left(a+\mu_m\cdot w\right)-\sin\alpha_1\left(w-\mu_m\cdot a\right)-\mu_m\cdot\rho_{ym}>0, \quad (12)$$

where μ_m, ρ_{ym} — theoretically maximum possible values of the friction coefficient and the friction radius.

From the technical specifications of any brake lining material, it is known that the friction coefficient μ, considered as a random variable, is defined within the range:

$$\mu=\bar{\mu}\pm\Delta\mu., \quad (13)$$

where Δμ – amplitude of the variation (spread) of the friction coefficient.

Taking equation (13) into account, the maximum friction value (μ_m) is assumed to be:

$$\mu_m=\bar{\mu}\pm\Delta\mu., \quad (14)$$

As noted earlier, experimental studies show that the contact in the "shoe–brake drum" friction pair is discrete in nature, meaning that the interaction between the friction material and the drum surface occurs unevenly and at discrete points. This fact is supported by numerous studies and observations conducted under both laboratory and practical conditions. Based on this, as well as the findings and developments presented in [15], the following assumptions can be made: the contact forces and the stress distribution are treated as random variables, which allows for a more accurate modeling of the brake shoe behavior under real operating conditions and for predicting its dynamic stability and the likelihood of self-excited vibrations, for example, as in [20, 21].

In this case, the maximum friction radius (ρ_{ym}) can be determined based on equation (15):

$$\rho_{ym}=\frac{r_b\cdot\beta_0\cdot k}{4\cdot\sin\left(\frac{\beta_0}{4}\cdot k\right)\cdot\cos\left(\frac{\beta_0}{4}\cdot(2-k)\right)}, \quad (15)$$

where r_b and β_0 are shown in Figure 2; k – a parameter defined as the ratio of the contour contact area in the friction pair (F_k) to the nominal area of the brake lining (F_H).

For the drum brake mechanism, it has been experimentally established that the parameter k can vary within the range of 0.21 to 0.48.

Figure 5 shows the calculation results for the brake mechanism of a heavy-duty vehicle with typical geometric parameters: β_0 = 120 deg; k = 0.21; r_b = 0.2 m and μ_m = 0.51.

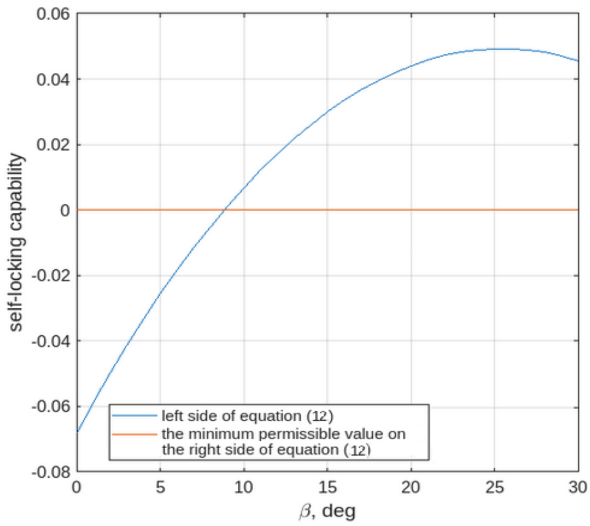


Figure 5: An example of a numerical solution of equation (12) for a heavy-duty truck.

From the analysis presented in Figure 5, a more detailed conclusion can be drawn regarding the operational characteristics of the drum brake mechanism under consideration. It becomes evident that, at relatively small values of the installation angle β (β < 10°), the system is prone to an increased risk of self-locking of the leading shoe. This phenomenon occurs as a result of an unfavorable orientation of the normal and tangential components of the contact force, which leads to an excessive increase in the frictional effect and, consequently, to a reduction in controllability of the braking process.

Such a state is typically accompanied by a pronounced instability of the braking torque and the occurrence of characteristic high-frequency vibrations, generally identified as brake squeal. These oscillations not only reduce driving comfort by producing undesirable noise and vibration but also negatively influence the operational reliability of the brake mechanism, accelerating wear of friction pairs and potentially reducing the overall service life of the system. From the standpoint of traffic safety, such instability is particularly undesirable, since it can impair the driver's ability to maintain predictable and effective braking.

As the value of the angle β increases, the unfavorable conditions for self-locking gradually diminish. In the intermediate range of 15° to 25°, the mechanism demonstrates significantly improved stability. Within this interval, the balance between the tangential and normal components of the contact interaction becomes more favorable, leading to a more uniform distribution of stresses across the frictional surfaces. As a result, the braking torque increases in a smoother and more predictable manner, minimizing the likelihood of oscillatory processes and enhancing both the efficiency and the safety of the braking action.

7. Results

Experimental studies, the results of which are presented in Figure 6, carried out in the laboratories of Kharkiv National Automobile and Highway University, have practically confirmed the validity and effectiveness of the proposed methodology for selecting the shoe support angle.

During testing of the front brake mechanism of a KAZ-4540 truck at small angles β, for example at β = 4 deg, self-excited vibrations accompanied by loud squealing and noticeable vibration were observed at almost all levels of air pressure in the brake chamber. After adjusting the design and increasing the support angle β to 15 deg, both in bench tests and in road tests, the squealing phenomenon completely disappeared, and the braking torque became more stable even under high loads. These results confirmed the necessity and feasibility of applying the proposed methodology in the design and refinement of brake systems for heavy-duty vehicles.

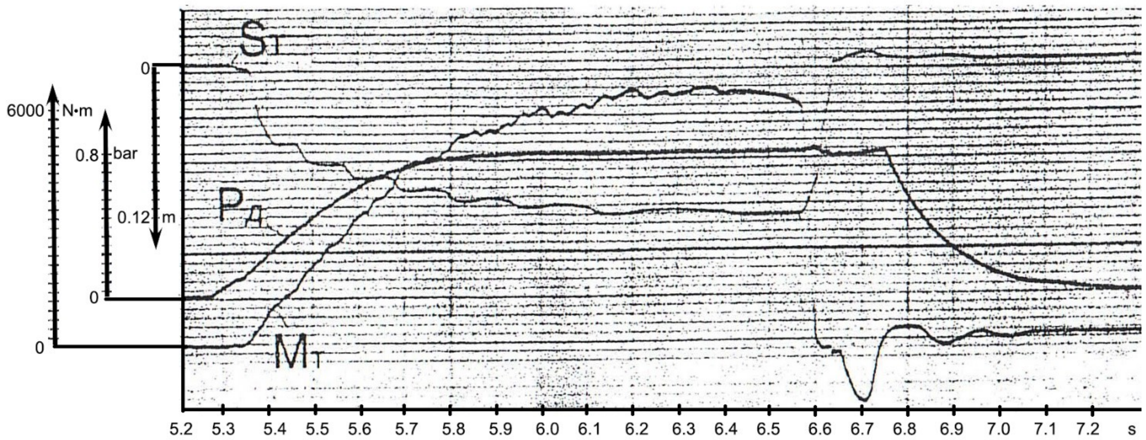
As for the trailing shoe, in real drum brake designs the probability that the denominator of equation (1) becomes less than or equal to zero is extremely low and can be practically disregarded.

This means that the occurrence of the self-locking effect for this shoe is virtually impossible, even under unfavorable operating conditions or with significant fluctuations in the coefficient of friction. Consequently, the risk of braking torque instability, as well as the associated self-excited vibrations and brake squeal, is minimal for the trailing shoe. This highlights the advantage of the trailing shoe design in terms of stability and reliability, especially in applications where consistent braking is critical.

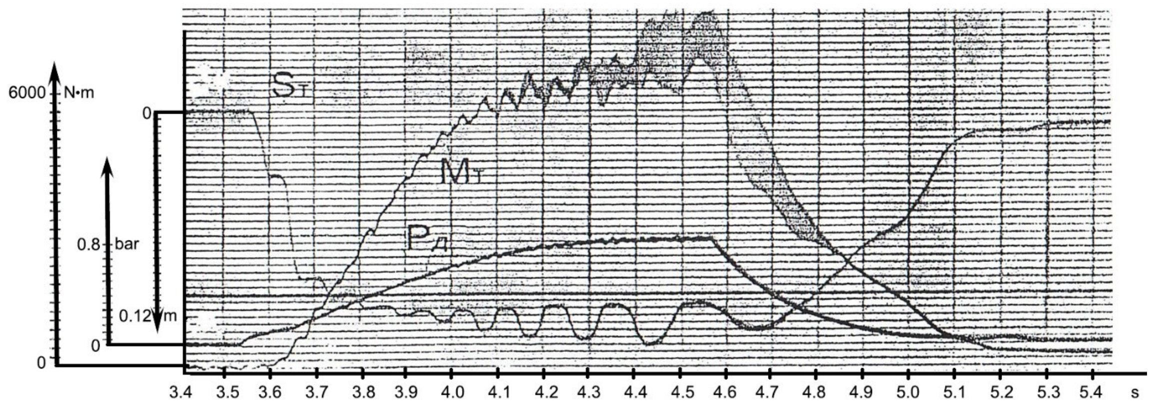
For this reason, it is not necessary to perform detailed calculations using equation (1) for the trailing shoe, nor to construct special graphical dependencies or carry out complex numerical simulations to assess the likelihood of self-locking.

The analysis of its operation can be limited to simpler engineering methods, such as calculating contact loads and checking the wear of friction linings.

When designing a drum brake mechanism, the main focus should be placed on the leading shoe, which largely determines the dynamic stability of the entire system.



a



b

Figure 6: Oscillogram of braking of a drum brake on a heavy-duty truck with a wedge-type expander and floating shoes: a) – braking without brake squeal; b) – with squeal; p – pressure in the pneumatic brake chamber; M_T – braking torque; S_T – displacement of the actuator push rod of the wedge-type expander mechanism.

8. Discussion

During the conducted study, it was established that the leading shoe plays a key role in the formation of the braking torque of a drum brake and, consequently, is the main source of potential instability and brake squeal. The obtained results are consistent with previously published data, which emphasized the importance of considering the geometric parameters of the shoe in the design process. However, in this study, emphasis was placed on the probabilistic nature of the parameters μ and ρ_y, which allowed for a more accurate description of the real behavior of the brake mechanism.

Particular attention should be given to the identified relationship between the installation angle of the shoe β and the stability of the braking torque. Selecting β within the range of 15°–25° proved optimal both in terms of preventing the self-locking effect and in reducing vibroacoustic effects.

These results confirm the practical value of the methodology and allow it to be recommended for use in the design of modern brake systems, especially for heavy-duty vehicles [22], where braking stability is critically important.

On the other hand, the trailing shoe exhibited a significantly lower tendency toward self-locking, which supports the validity of simplifying calculations for this system element. Nevertheless, its influence on braking dynamics should not be entirely neglected, particularly under high loads and with variations in the friction coefficient over time. Additional studies aimed at examining the distribution of contact stresses for both shoes could provide an even more comprehensive understanding of the brake mechanism's performance.

Finally, it should be noted that the developed approach can be further extended through the application of modern numerical modeling methods and machine learning techniques for the analysis of large experimental datasets.

This would allow for the consideration of nonlinear effects, shoe wear, and changes in material properties over time, improving the accuracy of predictions and enabling the design of more reliable and durable brake systems. Moreover, accounting for the influence of the random nature of contact parameters and the shoe geometry on the steady-state development of braking torque makes it possible to predict not only short-term self-excited vibrations but also the long-term dynamic stability of the system, which is critically important for the operation of heavy-duty vehicles.

9. Conclusions

Based on the conducted research, the following conclusions can be drawn:

The leading shoe is a critical element in the brake mechanism that generates noise. It largely determines the level and stability of the braking torque of a drum brake. In the design process, priority attention should be given to its geometric parameters and to ensuring conditions that eliminate the possibility of self-locking.

Optimizing the support angle of the leading shoe reduces vibroacoustic effects. Selecting the installation angle β within the range of 15 – 25 deg provides a stable distribution of contact stresses, reduces the likelihood of self-excited vibrations, and eliminates brake squeal. This has been confirmed by both bench and road tests.

Considering the random nature of contact parameters improves modeling accuracy. Applying a probabilistic approach to describe the friction coefficient μ and the effective friction radius ρ_y allows for a more accurate prediction of the system's dynamic stability and increases the reliability of the designed brake mechanisms.

References

[1] Gräbner, N., Schmid, D., & von Wagner, U. (2023). On Drum Brake Squeal-Assessment of Damping Measures by Time Series Data Analysis of Dynamometer Tests and Complex Eigenvalue Analyses. *Machines*. Volume *11*(12). pp. 1048. https://doi.org/10.3390/machines11121048

[2] Jacobsson, H. (2003). Aspects of Disc Brake Judder. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering.* Volume 217(6). pp. 419-430. https://doi.org/10.1243/095440703766518069

[3] Saad, A., Watany, M., Abouel-Seoud, S., Abdel-Gawad, I., & Society for the Advancement of Material and Process Engineering. (1998). Passenger Car Disc Brake Squeal. In *Society for the Advancement of Material and Process Engineering.* Volume 2. pp. 1158-1169.

[4] Nobari, A., Ouyang, H. and Bannister, P. (2015). Uncertainty quantification of squeal instability via surrogate modelling. *Mechanical systems and signal processing*. Volume 60-61. pp. 887-908. https://doi.org/10.1016/j.ymssp.2015.01.022

[5] Aviles, R., Hennequet, G., Hernandez, A. and Llorente, J.I. (1995). Low frequency vibrations in disc brakes at high car speed. *Part I: experimental approach. International Journal of Vehicle Design.* Volume 16:6, pp. 542-555. https://doi.org/10.1504/IJVD.1995.061920

[6] Hochlenert, D., Spelsberg-Korspeter, G., and Hagedorn, P. (2006). Friction Induced Vibrations in Moving Continua and Their Application to Brake Squeal. ASME. *J. Appl. Mech*. May 2007; Volume 74(3): 542–549. https://doi.org/10.1115/1.2424239

[7] Batrakova A.G., Gredasova O. (2016). Influence of Road Conditions on Traffic Safety, *Procedia Engineering*, Volume 134, pp. 196 – 204. https://doi.org/10.1016/j.proeng.2016.01.060

[8] Wang, X, John, S, & Ren, H. (2011). A Study of Squeal Noise in Vehicle Brake System. *Proceedings of the ASME 2011 International Mechanical Engineering Congress and Exposition. Volume 8: Mechanics of Solids, Structures and Fluids; Vibration, Acoustics and Wave Propagation*. Denver, Colorado, USA. November 11–17. pp. 711-716. https://doi.org/10.1115/IMECE2011-62084

[9] Nouby, M, Srinivasan, K. (2011). Simulation of the structural modifications of a disc brake system to reduce brake squeal. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*. Volume 225(5). pp. 653-672. https://doi.org/10.1177/2041299110394515

[10] Ouyang, H., Mottershead, J. E., and Li, W. (2003). A Moving-Load Model for Disc-Brake Stability Analysis. *J. Vib. Acoust*. Volume 125(1). pp. 53–58. https://doi.org/10.1115/1.1521954

[11] Ouyang, H., Mottershead, J. E., Cartmell, M. P., Brookfield, D.J. (1999). Friction-induced vibration of an elastic slider on a vibrating disc, *Elsevier BV*, Volume 41(3), pp. 325-336. https://doi.org/10.1016/s0020-7403(98)00059-9

[12] Fieldhouse, John D. and Newcomb, T.P. (1996). Double pulsed holography used to investigate noisy brakes*. Optics and lasers in engineering*, Volume 25(6). pp. 455-494. https://doi.org/10.1016/0143-8166(95)00094-1

[13] Rhee, S.K., Jacko, M.G., Tsang, P.H.S. (1991). The role of friction film in friction, wear and noise of automotive brakes. *Wear.* Volume 146(1). pp. 89-97. https://doi.org/10.1016/0043-1648(91)90226-K

[14] Nosseir, T.A., Said, M.A.R., Nahas, N.S. El and Fetouh G. A. E. (1998). Significance of squeal in disc brake design. *International Journal of Vehicle Design.* Volume 19(1). pp. 124-133. https://doi.org/10.1504/IJVD.1998.062098

[15] Bogomolov V., Leontiev D., Yaryta O., Frolov A., and Kostiennikov O. (2025). Stability of vehicle braking system output parameters and characteristics of utilized adhesion curves. *International Journal of Mechatronics and Applied Mechanics*, Volume 1(20), pp. 334-340. https://doi.org/10.17683/ijomam/issue20.34

[16] Heilig, J., Wauer, J. (2003). Stability of a Nonlinear Brake System at High Operating Speeds. *Nonlinear Dynamics*. Volume 34, 235–247. https://doi.org/10.1023/B:NODY.0000013506.20009.70

[17] Bogomolov, V., & Leontiev, D. (2025). On the distribution of normal reactions between the axes of a balancing bogie during vehicle braking. In *AIP Conference Proceedings.* Volume 3428(1), p.020003. https://doi.org/10.1063/12.0038594

[18] M Bulgakov, S Shuklynov, A Uzhva, D Leontiev, V Verbitskiy, M Amelin and O Volska (2020) Mathematical model of the vehicle initial rectilinear motion during moving uphill. 24th Slovak-Polish International Scientific Conference on Machine Modelling and Simulations - MMS 2019. *IOP Conf. Ser.: Mater. Sci. Eng.* 776:012022. https://doi.org/10.1088/1757-899X/776/1/012022

[19] Fedosov A.S. (1982). *Veroyatnostnyiy podhod k proektirovaniyu i raschetu barabannyih tormozov* [Probabilistic approach to design and calculation of drum brakes]. Automobile transport. Volume 19. 90-98.

[20] Kozhushko, A., Pelypenko, Y., Kravchenko, S., & Danylenko, V. (2023). Improving the Procedure for Modeling Low Frequency Oscillations of the Free Surface Liquid in a Tractor Tank. EasternEuropean Journal of Enterprise Technologies, 122(7), 61–68. https://doi.org/10.15587/1729-4061.2023.277254

[21] Kozhushko, A. (2023). Hydrodynamics Analysis on Partially Filled Agricultural Tanks by Driving Cycle of Transportation. *Lecture Notes in Networks and Systems*, Volume 762, 253–262. https://doi.org/10.1007/978-3-031-40628-7_21

[22] Kozhushko, A., Veretennikov, I., Pelypenko, Y., Gontar, Y., Trembach, O., Shulyak, M. & Yaryta, O. (2025). Investigating the influence form and intensity of the accelerating characteristic machine with a dual-flow non-reversible transmission. International Journal of Mechatronics and Applied Mechanics, Volume 1(19), pp. 225–232, 2025. doi: https://dx.doi.org/10.17683/ijomam/issue19.26.

[23] Dejian Meng; Cenxu Guan; Wenbo Li; Jiaming Liang; Lijun Zhang. (2025). Nonlinear dynamic modeling and analysis of steering brake squeal considering the deflection effect of friction pairs. *Nonlinear Dynamics*, Volume 236. 112971. https://doi.org/10.1016/j.ymssp.2025.112971

[24] Hugo Heidy Miyasato, Vinícius Gabriel Segala Simionatto, Milton Dias Junior (2026). Bounded responses from a nonlinear model with mode-coupling instability and negative stiffness in one of its contact interfaces. *International Journal of Non-Linear Mechanics.* Volume 180*, 105237. https://doi.org/10.1016/j.ijnonlinmec.2025.105237*

[25] Kim, S., Lee, S., Kim, S., & Nam, J. (2025). Influence of Brake Pad Temperature Variation on the Squeal Noise Characteristics of Disc's In-Plane Vibration Mode. *Sensors*, Volume *25*(13), 4080. https://doi.org/10.3390/s25134080

[26] Meehan, Paul A. and Leslie, Andrew C. (2021). On the mechanisms, growth, amplitude and mitigation of brake squeal noise. *Mechanical Systems and Signal Processing.* Volume 152. 107469 1-19. https://doi.org/10.1016/j.ymssp.2020.107469

[27] Zhu, Q., Xie, J., Zhang, W., Chen, G., & Tuo, J. (2023). Effect of the braking parameter on disc brake squeal of a railway vehicle. *Wear*, Volume *522*, 204884. https://doi.org/10.1016/j.wear.2023.204884

[28] Rodrigues R do N, Achtenová G, Gonçalves VV, de Araújo Bezerra R. (2025). A complex eigenvalue approach to brake squeal performance: Comparing commercial and Coconut shell-reinforced friction materials in vented railway brake discs. *Noise & Vibration Worldwide*. Volume 56(8-9). 458-473. doi:10.1177/09574565251348866

[29] Dias A, Rodrigues R, Bezerra R, et al. (2021). Light duty automotive duplex drum brake squeal analysis using the finite element method. *Proc IME C J Mech Eng Sci*; Volume 235(22): 6052–6068. https://doi.org/10.1177/09544062211008465

[30] Maciel MHC, Rodrigues RN, Costa CAS, et al. (2023). Parametric analysis on temperature influence on brake squeal generation in a single-seater off-road vehicle's disc brake. *Noise Vib Worldw*; Volume 54(10-11): 570–586. https://doi.org/10.1177/09574565231203250