International Journal of Mechatronics and Applied Mechanics
2025, Issue 22, Vol. I

MODELING OF WORKING PROCESSES OF THE CUTTING WITH A SINGLE GRAIN AT ABRASIVE GRINDING

Volodimyr Kalchenko [0000-0002-9072-2976], Andrij Yeroshenko [0000-0002-1629-9516],
Serhii Boiko [0000-0001-8341-6973], Pavel Ignatenko [0000-0002-0967-1631]
Chernihiv Polytechnic National University, 14027 Chernihiv, Ukraine
Email: svboyko.cstu@stu.cn.ua

Abstract - A mathematical model is proposed, which allows calculating the cutting force to the contact grinding-wheel and workpiece shapes including abrasive grain, its size, grain protrusion of the bond, the relative position of grains on the surface of the wheel and the depth of immersion grains in the material of the workpiece. Experimental studies of the accuracy of forming surfaces of rotary bodies during circular grinding based on 2 factors (rotation frequency of the workpiece and cutting depth) were carried out using the central composite plan. The optimization problem was solved, an optimal desirability profile was constructed.

Keywords: Working process of grinding, Contact patch of grinding wheel, Cutting force at grinding.

1. Introduction

Abrasive grinding process occupies a significant place in mechanical engineering as a method of finishing. During grinding in the contact with the workpiece of abrasive tool there is sliding, plastic deformation of the workpiece or cutting material, which is accompanied by considerable forces. With the increase of the forces the wear of abrasive tools is higher, the period of stability is reduced, the temperature in the cutting zone grows, and that sometimes leads to cracks and scorches on the treated surface. This increases the roughness of the surface finish and reduces the accuracy of processing. Therefore, the issue of studying the forces during abrasive grinding and the influence of various factors on their change is the subject of many studies.

The model describes the interconnection between input and output values of grinding. Currently, there are three approaches to modeling of cutting forces, namely: statistical, energetic and simulation-physical approaches.

The statistical approach [17], Tonshoff et al. [15] means the grinding process as cutting by several cutting edges. All grains on the wheel surface in contact with the workpiece material are supposed to cut at the same cutting depth. The total cutting force during grinding is the sum of all cutting forces on each active grain.

The energetic approach [17], Yanlong et al. [20] provides that grain plunging into the workpiece will pass 3 stages, namely: sliding, scratching, cutting, depending on the depth of the grains immersion in the workpiece. Thus, the total energy consumption is the sum of all 3 phases.

The concept of the simulation-physical approach [17] is based on physics of the grinding modeling. The starting point of the simulation is a numerical description of the wheel properties (Anderson et al. [1]), which can be established by measuring, analytical or mathematical modeling. Under this approach, the interaction (Kalchenko et al. [9]) of abrasive grains and the workpiece are investigated using the finite element method.

Forces of microcutting by a single grain in the diamond grinding were investigated by Grabchenko et al. [4], while by abrasive grinding - by Kalchenko et al. [8]. The authors of these works also conducted research of the microcutting process, but there are no mathematical models that can predict the cutting forces on the contact patch of the workpiece and the abrasive wheel, considering the form the abrasive grain, its size, grain protrusion of the bond, the relative position of grains on the wheel sutface and the immersion depth of grains in the material of the workpiece.

Among all cutting tools, the abrasive ones are the only ones where standards do not regulate the form of grains, their geometry, the law of grains distribution by shape, size, and position on the workpiece surface.

https://doi.org/10.17683/ijomam/issue22.v1.24

The shape of the abrasive grains is the basis for calculation of the number of cutting edges, the size of the cutting corners and the shape of the working part of the grain in contact with the workpiece. The shape and geometry of the grains are difficult and in most cases is uncertain.

Abrasive materials are grains of varying sizes and shapes, which are divided into fractions. The set of grains of the certain size dominating in the composition is called the main fraction. Therefore, the granularity of the material is determined by the grains of the main fraction. The number of possible models (Grabchenko et al. [4]) of distribution of elements of the working surface of the grinding wheel is big enough. The choice of any of the law of the distribution is determined by the relative characteristics of the distribution to the requirements of the research and adequate description of the experimental data obtained.

It should be noted that the current studies are characterized by the definite impact of individual parameters of the grinding wheel on the cutting forces. However, in actual conditions, the grinding is simultaneously influenced by factors, which cause cutting forces. In the existing literature sources, there are no options on calculating cutting forces on the contact patch of the wheel and the workpiece with the impact of the shape of the abrasive grain, the immersion depth of grains in the workpiece material, positions of grains on the working surface of the wheel, and the number of grains on the contact patch as a whole on the value of the cutting force.

The purpose of the paper is to develop a mathematical model that allows predicting the value of the cutting forces in the interaction of the wheel surface with the material of the workpiece.

2. Determination of the Abrasive Grains Shape

As mentioned in the sources Cong et al. [3], Kacalak et al. [6,7], the shape and geometry of the abrasive grains approximate to the simplified shapes subjected to mathematical description to reduce uncertainty of the shape and geometry of the abrasive grains.

Thus, to model the abrasive grains, let's choose a convex regular polygon – octahedron, shown in Fig. 1.

It is known that grinding materials are divided by grit size. The abrasive grit size depends on the percentage of the main fraction. It is indicted in Grabchenko et al. [5] that the grain composition of grinding materials is characterized by percentage of these fractions, including: limit, large, major, complex, and fine fractions.

In the literature Grabchenko et al. [5] it was observed that the grains composition of the grinding materials can be described by the law of normal distribution (1).

$$N(x,\mu_x,\sigma_x) = \frac{1}{\sigma_x\cdot\sqrt{2\cdot\pi}}\cdot\exp\left(-\frac{(x-\mu_x)^2}{2\cdot{\sigma_x}^2}\right), \qquad (1)$$

where,

x – current size of the abrasive grain; μx – average size of the abrasive grain; σx – average square deviation of the size of the abrasive grain.

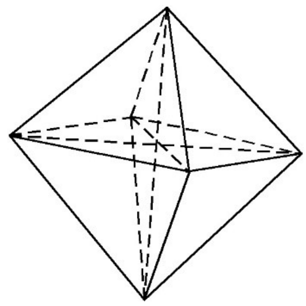

Figure 1: Simplified shape of the abrasive grain as an octahedron

According to the Grabchenko et al. [5] the size of the abrasive grains included in grinding materials may vary from x_{min} to x_{max} because according to the reference of formula (1) the probable grains availability from x_{min} to x_{max} is given by the expression (2):

$$P(x_{min} < x < x_{max}) = \int_{x_{min}}^{x_{max}}\frac{1}{\sigma_x\cdot\sqrt{2\cdot\pi}}\cdot exp\left(-\frac{(x-\mu_x)^2}{2\cdot{\sigma_x}^2}\right) \qquad (2)$$

where x_{min} – smallest size of the abrasive grain; x_{max} – maximum size of the abrasive grain.

The formula (2) allows calculating the probable grains availability in fractions of the abrasive material, for which the parameters of the normal distribution curve are known.

The outer surface of the wheel is a surface of the geometrically correct shape, outlined through the tops of the most protruding grains in the working layer of the tool (Fig. 2).

Bond surface is a surface of the geometrically correct shape, replacing with the closest approximation the actual bond surface in the innergrain layer of the tool.

Working layer of the wheel is a layer that is located between the outer surface of the wheel and the bond surface.

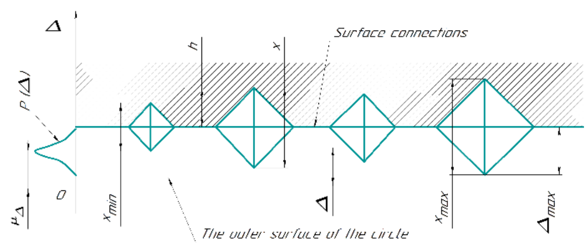

Figure 2: Scheme of the working layer of the grinding wheel

The thickness of the working layer of the tool Δmax can be determined experimentally, or approximately calculated using the formula (3) Grabchenko et al. [5].

$$\Delta_{max} \approx x_{max} \cdot (1-\varepsilon) = (\mu_x + 3 \cdot \sigma_x) \cdot (1-\varepsilon) \quad (3)$$

where ε – coefficient that considers the grains immersion in the bond (4).

$$\varepsilon = \frac{h}{x} \quad (4)$$

where h – immersion depth of the grain in the bond; x – the size of the abrasive grain.

Experimental determination of ε is carried out by measuring the largest protrusion of the grains in the wheel of grains (by means profilometry of the working surface), for the wheel of this grit size, using formula (3), the formula (5) can be calculated:

$$\varepsilon = 1 - \frac{\Delta_{max}}{\mu_x + 3 \cdot \sigma_x} \quad (5)$$

Any specific value of the grain protrusion Δ is random. Therefore, distribution Δ within 0 to Δ_{max} can be described by the law of normal distribution (6) with sufficient reliability Grabchenko et al. [5].

$$N(\Delta, \mu_\Delta, \sigma_\Delta) = \frac{1}{\sigma_\Delta \cdot \sqrt{2 \cdot \pi}} \cdot \exp\left(-\frac{(\Delta-\mu_\Delta)^2}{2 \cdot {\sigma_\Delta}^2}\right) \quad (6)$$

with average square deviation (7):

$$\sigma_\Delta \approx \frac{\Delta_{max}}{6} = \frac{1}{6} \cdot (\mu_x + 3 \cdot \sigma_x) \cdot (1-\varepsilon) \quad (7)$$

and the grouping center (8):

$$\mu_\Delta \approx \frac{\Delta_{max}}{2} = \frac{1}{2} \cdot (\mu_x + 3 \cdot \sigma_x) \cdot (1-\varepsilon) \quad (8)$$

Therefore, in accordance with (6) the probability of the grain protrusion within 0 to Δ_{max} from the bond is determined by the expression (9):

$$P(0 < \Delta < \Delta_{max}) = \int_0^{\Delta_{max}} \frac{1}{\sigma_\Delta \cdot \sqrt{2 \cdot \pi}} \cdot exp\left(-\frac{(\Delta-\mu_\Delta)^2}{2 \cdot {\sigma_\Delta}^2}\right) d\Delta \quad (9)$$

Abrasive grains can take any position on the working surface of the wheel, so to ease the problem, we consider only the relative position of the grains related to other grains, ignoring the angular position of the abrasive grains (Fig. 3).

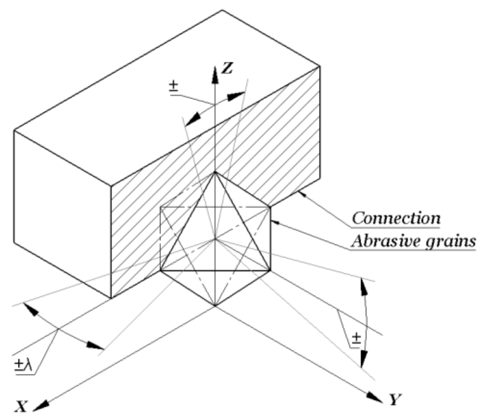

Figure 3: Angular position of the abrasive grain

All possible positions taken by the grains can be divided into 3 types:

– Grains arranged "one after the other";

– Grains with "partial overlap of one grain by the other";

– Grains that are "located nearby".

From the above, it is clear that the possible positions of the abrasive grains on the surface of the wheel are subjected to the two-dimensional distribution law.

Let us assume that the positions of the grains in the direction of *OX* axis (L - circumference, which has a diameter D) and *OY* axis (B - height of the grinding wheel) are not correlated, then the correlation coeffcient $r_{xy} = 0$.

In other words, the probable availability of grains in a certain position on the working surface of the grinding wheel equals the product of the probabilities toward the axes *OX* and *OY*, i.e., the probability towards the axes independent between each other.

3. The Law of the Grains Distribution on the Working Surface of the Wheel

The positions of the grains on the working surface of the wheel can be described by the uniform distribution law. The two-dimensional uniform distribution law with the correlation coefficient $r_{xy} = 0$ has the form (10):

$$N(x,y) = \begin{cases} \frac{1}{(L-0)\cdot(B-0)}, & at\ 0<x<L,\ 0<y<B \\ 0, & at\ (x,y) \notin ABCD \end{cases} \quad (10)$$

where *ABCD* – rectangle with vertices A*(0; 0), B(B; 0), C(B; L), D(0; L)*.

The positions of the abrasive grains on the working surface of the wheel can be changed in the direction of axis *OX* (from *0* to *L*) and *OY* (from *0* to *B*), because according to (10) the probability of the grain availability in a certain place on the working surface of the grinding wheel is given by (11):

$$P(x,y) = \int_0^L \int_0^B N(x,y)dxdy \quad (11)$$

Not all grains that we can see on the surface of the wheel are involved in removing the workpiece material. Only those grains are active (Fig. 4) that at some time have the shape Δ, less than the thickness of inequality k on the contact surface, which occurs as a result of the actions of the previous grains.

The values of Δ and k are deducted from any zero surface, in this case (see. Fig. 4), they are deducted from the outer surface of the wheel.

The surface of the workpiece is a surface of the geometrically correct shape, outlined through the tops of the most fallen out irregularities on the workpiece.

Thus, the condition for participation in the grain in the activity is a positive value of the immersion value of the abrasive grains (12) in the workpiece material Grabchenko et al. [5]:

$$a_z = k - \Delta \tag{12}$$

where k – coordinate of the top of the irregularity of the workpiece material; Δ – protrusion of the abrasive grain on the surface of the wheel.

If the the value a_z lies between 0 and k, then the interaction of the grain with the workpiece material takes place, so:

– With $a_z < 0$ – the grain is not involved in the work;

– When $a_z = 0$ – the grain slides on the workpiece material;

– With $a_z > 0$ – the grain is involved in the work, plastically deforming or cutting material piece.

In the process of cutting by the abrasive grain, the equality is fair (13):

$$\frac{a_z}{\rho} > 0.1 \tag{13}$$

where a_z – value of the abrasive grain immersion in the workpiece material; ρ – value of the rounding of the top of the abrasive grain.

If the value of the equality (13) is less than 0,1 – only the plastic deformation takes place, and if more than 0,1 – cutting of the workpiece material by the grain takes place.

As noted earlier, the abrasive grain has some immersion h in the bond (see. Fig. 2), expressed by the coefficient ε according to (4) and should have some critical value $[\varepsilon]$, which is expressed by (14):

$$\varepsilon > [\varepsilon] \tag{14}$$

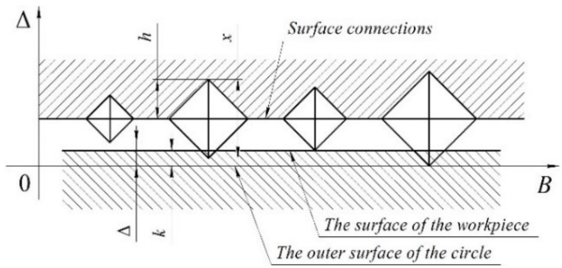

Figure 4: Interaction of the wheel surface with the workpiece surface

The expression (14) is explained by the fact that the forces acting on the abrasive grain when cutting, will try to snatch the grain out of the bond, so the value of the coefficient ε cannot be less than the critical one, since the grain will be snatched out of the bond and will fall out both the cutting zone and the abrasive wheel itself.

To determine the cutting forces arising in the contact zone of the abrasive wheel and the workpiece, we need to determine the total number of the abrasive grains, located in the surface layer (Fig. 5) of the abrasive wheel.

As seen in Fig. 5, the surface layer with thickness H is the amount of the working layer Δmax and the layer which equals the maximum size of the abrasive grain in the composition of the grinding material.

The internal volume of the grinding wheel is characterized as follows: grit, hardness, structure; and these parameters depend on the volume of the grains composition and the bond. These options depend on the volume composition of the grains β_V, the bond β_C, porosity of the wheel β_P (15) Grabchenko et al. [5]:

$$\beta_V + \beta_C + \beta_P = \frac{V_V}{V_K} + \frac{V_C}{V_K} + \frac{V_P}{V_K} = 1 \tag{15}$$

where V – volume.

In accordance with the above, the volume of the surface layer of the abrasive wheel is determined by the formula (16):

$$V_{SL} = \frac{\pi \cdot D^2 \cdot B}{4} - \frac{\pi \cdot (D - 2 \cdot H)^2 \cdot B}{4} = \pi \cdot B \cdot H \cdot (D - H) = \pi \cdot B \cdot (x_{max} + \Delta_{max}) \cdot (D - (x_{max} + \Delta_{max})) \tag{16}$$

The relative content of grains, the bucns, and pores in the abrasive wheel can be identified under Grabchenko et al. [5]. Thus, the total amount of the abrasive grains in the surface layer of the wheel (17):

$$V_V = V_{SL} \cdot \beta_V \tag{17}$$

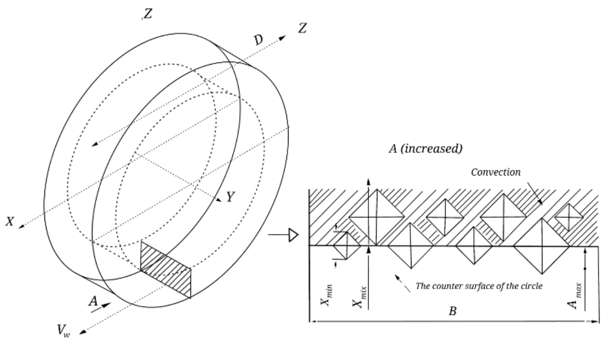

Figure 5: Scheme of the surface layer of the abrasive wheel

Since we have adopted a simplified shape of the abrasive grains as an octahedron (see. Fig. 1), its volume can be determined by formula (18), and considering the probability of (2), accordingly the volume of each grain is determined by (19):

$$V = \frac{x^3}{6} \tag{18}$$

$$V(x_{min} < x < x_{max}) = \frac{P(x)^3}{6} \tag{19}$$

Therefore, according to (17) and (19), the probable number of the abrasive grains in the surface layer is defined by the expression (20):

$$Z = \frac{V_V}{\sum_{i=x_{min}}^{x_{max}} V(i)} \tag{20}$$

Contact zone of the abrasive wheel and the workpiece are characterized by the patch surface that is calculated for the flat grinding by the wheel periphery (21):

$$S_{ZC} = L_P \cdot B \tag{21}$$

where L_P – length of the contact arc of the abrasion wheel and the workpiece at the flat grinding by the wheel periphery; B – height of the abrasive wheel.

The length of the contact arc is determined by (22) within the contact angle (23) Kalchenko et al. [8], Grabchenko et al. [5]:

$$L_P = \left(1 \pm \frac{V_D}{60 \cdot V_W}\right) \cdot \sqrt{D \cdot t} \tag{22}$$

$$\psi_P \approx 2 \cdot \sqrt{\frac{t}{D}} \tag{23}$$

where V_D – speed of the forward motion of the workpiece, m/min; V_W – speed of the rotational motion of the abrasive wheel; t – cutting depth; D – diameter of the grinding wheel; ψ_P – contact angle of the grinding wheel and the workpiece.

"+" sign is taken at the counter, and "–" at the passing direction of the grinding wheel and the workpiece.

Thus, in the contact zone a definite number n of the abrasive grains occur, therefore cutting forces are determined by formulas (24) and (25) as the sum of the forces on active grains:

$$P_Z^P = \sum_{i=1}^{n} F_{Zi} \tag{24}$$

$$P_Y^P = \sum_{i=1}^{n} F_{Yi} \tag{25}$$

where Fz – cutting force on the single active grain toward the axis OZ; Fy – cutting force on the single active grain toward the axis OY; n – the number of active grains in the contact zone.

There are many obstacles to experimentally confirming the modeling performed. Direct measurement of cutting forces with a single grain is not possible due to the peculiarities of the abrasive grinding process. Determining the total forces is complicated and often done indirectly, for example, by measuring the power of the cutting process. So, the authors decided to do an experimental study of the processing results (roughness of the processed surface) by controlling the cutting modes.

Determining the total cutting forces (24-25) will allow optimizing the cutting mode by setting the cutting speed and depth (see Table 1). Next, an experimental study was conducted to determine the effect of the specified cutting modes on the roughness of the machined surface.

4. Experiment

4.1 Equipment and Materials used during the Experiment

1. The cylindrical grinding machine 3M151 model is designed for external grinding of smooth and intermittent cylindrical surfaces and shallow conical surfaces by longitudinal and plunge grinding methods.
2. Tool – grinding wheel 25AF46 GOST 2424-83 (white electrocorundum, grain size – 40 microns, on the ceramic bond, medium softness, operating speed is 50 m/s).
3. Lubricating and cooling medium – "CIMTECH"D18-02; Batch: 1711-07A (synthetic fluid for low-intensity processing of ferrous metals by cutting and grinding, intended for use both on individual machines and in central systems).
4. Workpiece – shaft Ø 15 mm, made of 41Cr4 steel (structural steel, technical requirements EN 10277 - products with improved surface treatments), without preliminary heat treatment.

The process of conducting the experiment is shown in Figure 6, the results of the experiments are shown in Table 1.

Figure 6: The process of the experiment

Table 1. Result of the experiments

Experiment number	Workpiece rotation speed, rpm	Cutting depth, mm	Roughness Ra, μm
1	110	0,02	1,25
2	110	0,07	2,5
3	150	0,02	1,25
4	150	0,07	2
5	102	0,05	1
6	158	0,05	2
7	130	0,015	0,63
8	130	0,085	1
9	130	0,05	**0,63**

4.2 Statistical Processing of Results

Figure 7 shows a graph of the interaction between the rotation frequency of the processed workpiece and the depth of the cutting during cylindrical grinding. The worst surface roughness is obtained when grinding at a low speed of the workpiece with a large depth of the cutting, till grinding takes place at the high speed of the workpiece.

4.3 Solving the Optimization Problem

The optimization problem has the following formulation: increasing the quality index of the machined surface (reducing the roughness Ra) based on desirability functions.

The procedure used in the treatment usually includes two stages:

1. Predicting reactions to dependent or *Y*-variables by fitting observed product characteristics using a regression equation based on the levels of independent or *X*-variables.

2. Determining the levels of the *X* variables that simultaneously yield the most desired predicted responses to the *Y* variables.

The challenge is to find the workpiece speed and the depth of the cutting levels that will produce the most desirable results of the surface quality.

To construct the desirability function, the values of the desirability index d are plotted on the ordinate axis, which vary from 0 to 1, with 0 corresponding to the worst value of the index, and 1 to the best one.

Figure 8 shows the optimal desirable profile. The figure consists of two line graphs. The graph in the the upper right corner displays the desirability function. The graphs of the upper line, in addition to the desirability function, display edges of the fitted dependence function (roughness in terms of Ra) on the corresponding dependent variable when fixing all other variables at optimal levels.

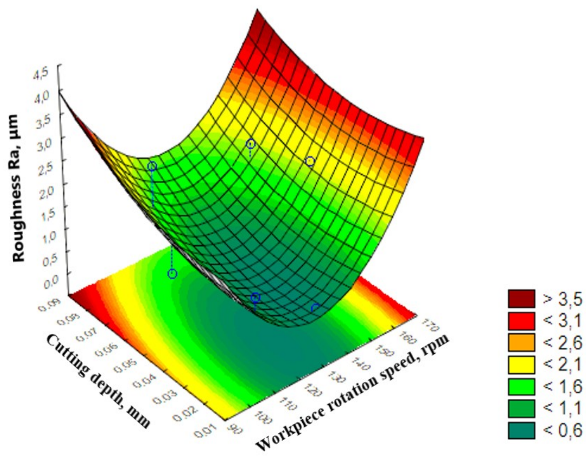

Figure 7: Graph of the interaction between the rotation frequency of the workpiece and the cutting depth during the cylindrical grinding

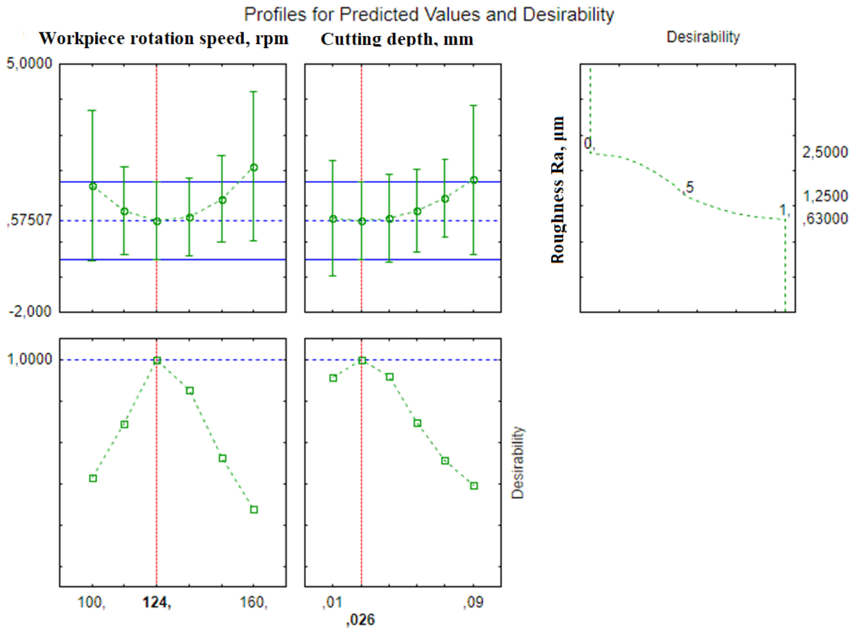

Figure 8: Optimal desirability profile

The optimal levels of the independent variables are shown in the graphs at the bottom of the figure by red lines. The same series of graphs shows changes in the desirability function when varying the corresponding independent variables.

As a result of the analysis of the obtained profile, the roughness value 0,57 μm was obtained, and the desirability value is 1. This value of the quality of the treated surface is achieved at a workpiece rotation speed n = 124 rpm and a cutting depth t = 0.026 mm. In practice, for convenience, the cutting mode indicators should be rounded to integer values: n = 125 rpm, t = 0.025 mm.

The results obtained correlate with the studies presented in [8].

5. Conclusions

The mathematical model was obtained on the dependence of the cutting force of the contact patch of the chosen shape of the abrasive grain, grain sizes, the size of the grain protrusion of the bond, the relative position of grains on the surface of the wheel and the depth of the grains immersion in the workpiece material. The directions for further research are considering the temperature on each active grain and on the contact patch in the whole with further optimized grinding process.

Experimental studies of the accuracy of forming surfaces of rotary bodies during cylindrical grinding based on 2 factors (rotation frequency of the workpiece and the cutting depth) were carried out using the central composite plan.

According to the results of experimental studies, processing of the statistical data was carried out to evaluate correlation residues and construct graphs of the interaction of the studied factors.

The optimization problem was solved, the optimal desirable profile was constructed, as a result of the analysis of which we have the roughness parameter value of 0.57 μm, at n = 125 rpm, t = 0.025 mm.

References

[1] Anderson D., Warkentin A., Bauer R. (2011), Experimental and numerical investigations of single abrasive-grain cutting, International Journal of Machine Tools & Manufacture, 51, 898-910.

[2] Chi Y., Li H. (2012), Simulation and analysis of grinding wheel based on Gaussian mixture model, Frontiers of Mechanical Engineering, 7(4), 427-432.

[3] Cong S., Yansheng D., Dongxue L., Shichao X.. (2018), Modeling and predicting ground surface topography on grinding chatter, Procedia CIRP, 71, 364-369.

[4] Grabchenko A., Fedorovich V., Pyzhov I., Kundrák J. (2014), 3D simulation of vibrating diamond grinding. Manufacturing Technology, 14(2), p. 153-160.

[5] Grabchenko A.I., Kalchenko V.I., Kalchenko V.V. (2016), Grinding with crossed axes of tool and workpice (in Russian), monograph. 2nd edition, revised, Chernihiv: CHNTU, 505 p.

[6] Kacalak W., Szafraniec F., Lipiński D. (2018), Methods for modeling the active surface of grinding wheels, Mechanik, 91 (10), 907-914.

[7] Kacalak W., Lipiński D., Szafraniec F., Tandecka K. (2018), The methodology of the grinding wheel active surface evaluation in the aspect of their machining potential, Mechanik, 91 (8-9), 690-697.

[8] Kalchenko V., Yeroshenko A., Boyko S., Sira N. (2017), Determination of cutting forces in grinding with crossed axes of tool and workpiece, Acta Mechanica et Automatica, 11(1), 58-63.

[9] Kalchenko V., Kalchenko V., Sira N., Yeroshenko A., Kalchenko D. (2020) Three-Dimensional Simulation of Machined, Tool Surfaces and Shaping Process with Two-Side Grinding of Cylindrical Parts Ends. In: Tonkonogyi V. et al. (eds) Advanced Manufacturing Processes. InterPartner 2019. Lecture Notes in Mechanical Engineering. Springer, Cham, 2020, p. 118-127.

[10] Kalpana K., Arunachalam N. (2018), Grinding wheel redress life estimation using force and surface texture analysis. Procedia CIRP, 72, 1439-1444.

[11] Grabchenko A.I., Mamalis A.G., Fedorovich V.A., Romashov D.V. (2016), Improving the design of diamond wheel for high-speed grinding. Journal of Machining and Forming Technologies. Nova Science Publishers, Inc. Volume 8, Number 1-2, 12 p.

[12] Mikhailets V.A., Chekhanova G.A. (2015), Limit theorems for general one-dimensional boundary-value problems. Journal of Mathematical Sciences, Vol. 204, No. 3, p. 333-342.

[13] Mikhailets V.A., Pelekhata O.B. (2018), Limit theorems for the solutions of boundary-value problems. Ukrainian Mathematical Journal, Vol. 70, p. 216-223.

[14] Shakhbazov Y., Shyrokov V., Fedorovych V. (2019), Specifying the Process Parameters for

Diamond Dressing of Grinding Wheels. Journal of Superhard Materials, Volume 41, p. 272–277.

[15] Tonshoff H.K., Friemuth T., Becker J.C. (2002) Process monitoring in grinding. CIRP Annals – Manufacturing Technology. Vol. 51, no. 2. P. 551 – 571.

[16] Uhlmann E., Koprowski S., Weingaertner W.L., Rolon D.A. (2016), Modelling and Simulation of Grinding Processes with Mounted Points: Part II of II - Fast Modelling Method for Workpiece Surface Prediction. Procedia CIRP, 46, 603-606.

[17] Xuekun Li. (2010). Modeling and simulation of grinding processes based on a virtual wheel model and microscopic interaction analysis: a dissertation for the degree of doctor of philosophy in manufacturing engineering. Worcester polytechnic institute, 145 p.

[18] Yanlong C., Jiayan G., Bo L., Xiaolong C., Jiangxin Y., Chunbiao G.. (2013), Modeling and simulation of grinding surface topography considering wheel vibration. The International Journal of Advanced Manufacturing Technology, 66(5–8), 937-945.

[19] Yan L., Rong Y.M., Jiang F., Zhou Z.X. (2011), Three-dimension surface characterization of grinding wheel using white light interferometer. International Journal of Advanced Manufacturing Technology, 55, 133-141.

[20] Yanlong C., Jiayan G., Bo L., Xiaolong C., Jiangxin Y., Chunbiao G.. (2013), Modeling and simulation of grinding surface topography considering wheel vibration. The International Journal of Advanced Manufacturing Technology, 66(5–8), 937-945.