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MULTI-CRITERIA KINEMATIC OPTIMIZATION OF A DOUBLE WISHBONE SUSPENSION MECHANISM

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Abstract - This paper deals with the multi-objective kinematic optimization of a double wishbone suspension mechanism used for race cars. The optimization purpose is to keep the contact patch area constant, by reducing the camber angle variation, to keep the steering behavior the same, by reducing the castor angle, and to reduce the slip angle by decreasing the bump steer. The study uses a two steps tuning approach based on the fact that in a double wishbone suspension system, the steering rod has a reduced influence on the camber and castor variation and the main control arms have small effects to the bump steer. So the camber and castor metrics are tuned in the first step and the bump steer is tuned in the second tuning step. For both tuning steps, the virtual model has the chassis rigidly mounted to the ground and the imposed motion was set to the wheel (+/-13mm), as we are interested in tuning the "on center" behavior of the suspension system. The multi body system study was performed using MSC.ADAMS (namely ADAMS/View - to develop the kinematic model of the suspension system, and ADAMS/Insight - to configure and perform the optimization process). Finally, an in-house made app (S&KSP - Static and Kinematic Suspension Parameter) based on MATLAB was developed for the mutual validation of the results.

Keywords: Front suspension, Multi-body system, Regression function, Kinematic optimization.

1. Introduction

It is well known that, any suspension system is designed so that it creates the best dynamic behavior of the vehicle. This behavior may vary from vehicle to vehicle, as different cars have different scopes although they use the same suspension mechanism. Any dynamic behavior can be divided into different design objectives (targets) and most of them are directly affected by the kinematic movement of the suspension mechanism. The dynamic targets can be tuned by optimizing different kinematic metrics.

Although most of the suspension systems have a wheel travel range in the vicinity of +/-80mm, in this range the kinematic behavior is hard to tune and understand, as at maximum travel the bushes, springs, dampers, control and chassis stiffness need to be taken into account [13-15].

The kinematic analysis and optimization of the double wishbone suspension mechanisms is a permanent concern and challenge. Important publications reveal a growing interest in analysis methods for multi-body systems (MBS) that may facilitate the self-formulating algorithms [1-4].

The main objective of the kinematic tuning is the "on center" behavior of the suspension system, as the full wheel travel is affected by bushes, springs, dampers and material deflections. The kinematic model does not take into account the body masses and the elastic & damping elements of the suspension system. The "on center" suspension behavior is given by several metrics, as: roll center height; castor center height; contact patch migration but the most important ones are camber, castor and bump steer angle variations, as most of the other metrics are influenced by this three.

The model is built of two control arms (upper and lower control arm) each are linked to the chassis in two points through two ball joint type joints and at the wheel carrier end with a ball joint, a steering rod which is linked to the wheel carrier with a ball joint and at the other end to the steering rack with another ball joint, a wheel carrier and a wheel that is linked to the wheel carrier with a rotational joint) (Fig. 1); such a layout is often referred as double wishbone (or SLA – Short-Long Arm) suspension mechanism [6, 9, 10, 14].

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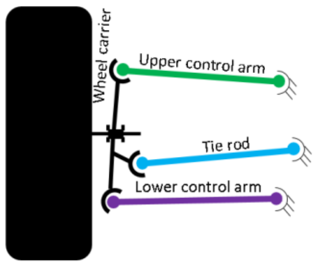

Figure 1: Double wishbone suspension layout

The optimization is approached through a screening experimental design. Based on the work space of the experiment, a linear regression model is developed, which is used to establish a relationship between the design variables (chassis hard points) and the design objectives (RMS of the camber, castor and bump steer variations). Although all the suspension systems have different targets set for the measured wheel angle variations, based on the specific use (track cars, off road cars) this paper will go through the tuning steps and try to reduce all the angle variations to zero. The application is for the suspension mechanism of the front wheels of a race car (Formula Student type).

2. The Optimization Algorithm

The most used software solutions in a virtual prototyping platform are CAD - Computer Aided Design, MBS - Multi-Body Systems, and FEA - Finite Element Analysis [1, 7]. The main component of the platform is the MBS software, which is used for analyzing, optimizing and simulating the kinematic behavior. The solid model of the system is created using CAD software and the geometry of the parts can be exported from CAD to MBS using standard format files, such as STEP or PARASOLID.

In the presented study, the locations of the suspension hard points are used to parameterize the mechanical system (Fig. 2). The model is created so that any change in the position of these points will affect the suspension arms and so resulting in a change in suspension behavior. The tuning presents the issue of a measured metric to achieve a given target by changing the design variables, while satisfying various design constraints.

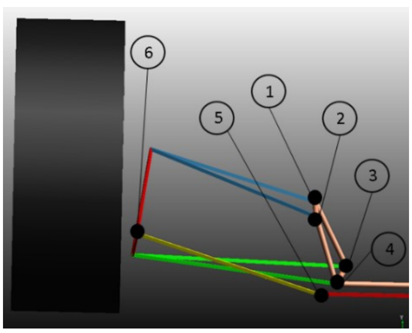

Figure 2: The design points

Based on the fact that any suspension system is symmetrically disposed relative to the longitudinal axis of the car, this paper will only tune a single part of a front double wishbone suspension system mechanism. In theory, in case of a double wishbone suspension system, the upper and lower control arms of the designed suspension system mainly control the vertical wheel motion and the camber and castor angles, while the track rod (steering arm) mostly affects the bump steer.

For the first step of the tuning process, the optimization is performed by using design of experiments (DOE) and the D-Optimal with Interactions regression technique (Fig. 3), in the following sequence: modeling the design objectives (minimize the camber and castor variations), set the design variables (Pt_1 to Pt_4 x,y,z coordinates), set the investigation strategy, and plan a set of trials in which the design variable value varies from one trial to another, execute the runs and record the performance of the system at each run, fit the results to a regression function, evaluate the goodness of fit, and optimize the system [5, 12].

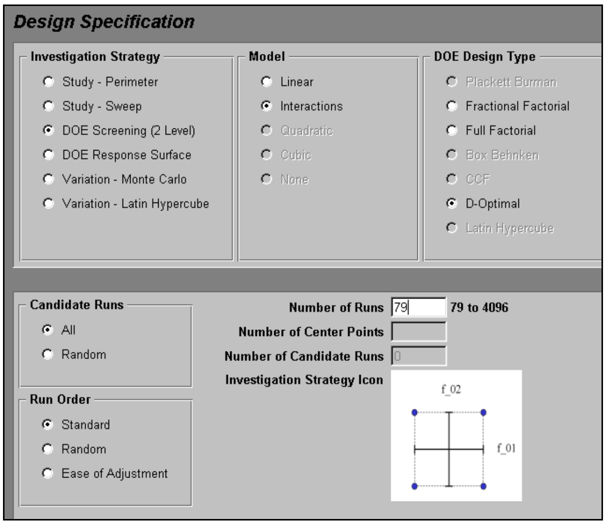

Figure 3: Design specifications for camber & castor tuning

The optimization study is performed with ADAMS/Insight, which is a powerful design of experiments software, providing a collection of statistical tools for analyzing the results of the experiments (trials). For the suspension system in study, the results, in terms of goodness of fit, have been obtained for the DOE Screening strategy with D-Optimal design and Interactions model. DOE Screening identifies the design variables and combinations of design variables that most affect the behavior of the system, picking the high and low values of a setting range. The D-Optimal design produces a model that minimizes the uncertainty of coefficients, consisting of a random collection of rows from a larger pool of candidates that are selected using minimization criteria [5, 12].

Table 1. Design space for camber and castor optimization

Trial	DV_1	DV_2	DV_3	...	DV_10	DV_11	DV_12
Trial 1	-1	1	-1	...	-1	-1	-1
Trial 2	-1	1	-1	...	-1	-1	-1
Trial 3	-1	1	-1	...	-1	-1	-1
...	...	...	...	...	...	...	...
Trial 77	-1	-1	1	...	-1	-1	-1
Trial 78	-1	-1	-1	...	1	1	-1
Trial 79	-1	1	1	...	-1	1	-1

Table 2. Several trials from the work space of the camber and castor experiment

Trial	DV_1	DV_2	DV_3	...	DV_10	DV_11	DV_12	Camber angle	Castor angle
Trial 1	-510	160	410	...	490	50	80	*0.05394*	*0.28035*
Trial 2	-510	160	410	...	490	50	80	*0.12576*	*0.17567*
Trial 3	-510	160	410	...	490	50	80	*0.12139*	*0.18534*
...	...	...	...	...	...	...	...	...	...
Trial 77	-510	80	490	...	490	50	80	*0.00252*	*0.16373*
Trial 78	-510	80	410	...	570	130	80	*0.13171*	*0.15523*
Trial 79	-510	160	490	...	490	130	80	*0.00351*	*0.1692*

Based on the design specifications, the design space and the work space of the experiment were created, considering the minimum number of runs/trials for the selected investigation strategy (in this case, 79 trials). The design space is a matrix with the rows representing the runs, and the columns representing the design variables settings, which are in a normalized representation (Table 1). The work space is a matrix with the rows indicating the trials and the columns identifying the design variables settings and resulting objectives values. There are combinations with the minimum and maximum values of the design variables, in accordance with the specific variation fields (e.g. for DV_1, the standard value is -470 [+/-40mm], so the variation field will be [-430, 510]).

For each trial, a simulation will be performed. After ADAMS/View completes the runs, the simulation results appear in the work space. As instant, a part from the work space is shown in Table 2. The analyses have been performed considering the +/- 13 mm wheel travel imposed motion (to define the “on center” suspension behavior).

Based on the work space of the experiment, an appropriate regression model (function) is developed, which is then used to establish a relationship between the design objectives (responses) and the variables (factors).

The regression function captures the factors - response relationships to a specified order (linear, quadratic or cubic), the best results (in terms of goodness of fit) being obtained for a linear model with interactions in this case (the interactions effects are captured through special terms that consist of products of design variables), defined as follow:

$$\begin{aligned} r\_0i &= a1 + a2 \cdot DV\_1 + a3 \cdot DV\_2 + a4 \cdot DV\_3 + \\ & \ldots + a13 \cdot DV\_12 + a14 \cdot DV\_1 \cdot DV\_2 + a15 \cdot \\ & DV\_1 \cdot DV\_3 + \ldots + a78 \cdot DV\_11 \cdot DV\_11 + \\ & a79 \cdot DV\_11 \cdot DV\_12 \end{aligned} \quad (1)$$

where: r_0i - the design objective (camber variation, castor variation, bump steer), DV_1 to DV_12 - the design variables, a1 to a79 - the coefficients computed by the regression analysis (a1 being the constant term), e - the remaining error that is minimized by the regression analysis.

The goodness of fit (Figures 4 and 5) is defined by the following statistical measures: R-squared (R2), Rsquared adjusted (R2adj), regression significance (P), range-to-variance (R/V), and F-ratio (F). Rsquared indicates the variance in the predicted results versus the real data. This is the proportion of total variability in the data which is explained by the regression model, a score of "1" indicating a perfect fit. R-squared-adjusted is similar to R-squared, but it is adjusted to account for the number of terms. Regression significance indicates the probability that the fitted model has no useful terms, small values, less than "0.01", indicating that the fit does have useful terms. Range-to-variance ratio indicates how well the model predicts values at the data points. F-ratio is used in the regression to test the significance of the regression, high values suggesting that the regression is significant [5, 12].

Goodness-of-fit for model

	Cam	Cast
R2	1	1
R2adj	1	1
P	0	0
R/V	1e+20	1e+20

Figure 4: Goodness-of-fit summary for the castor and camber responses

Fit for regression "Cam"

	DOF	SS	MS	F	P
Model	78	0.329	0.00422	1e+20	0
Error	0	1.77e-30	0		
Total	78	0.329			
R2	1				
R2adj	1				
R/V	1e+20				

a.

Fit for regression "Cast"

	DOF	SS	MS	F	P
Model	78	0.63	0.00807	1e+20	0
Error	0	2.2e-29	0		
Total	78	0.63			
R2	1				
R2adj	1				
R/V	1e+20				

b.

Figure 5: Fit regression for camber angle (a) and castor angle (b)

ADAMS/Insight provides graphical indicators that indicate the soundness of the results, as follows: green - entity is likely appropriate; yellow - entity is not necessarily bad, but could use some consideration; red - entity needs investigation.

The fit to regression results shown in Fig. 5 indicate that the regression models for the selected strategy (DOE Screening, D-Optimal design, Interactions model) matches the test data very well, for all design objectives. The fit table contains also the number of independent variables that go into the estimation of a parameter (DOF), the sum of squares (SS), and the mean square (MS), for the three parts of the statistical model - regression (model), residual (error), and total [5, 12].

In the final step, the effective optimization of the suspension mechanism has been performed for minimizing the root mean squares of the design objectives. The method used in optimization is OptDes-GRG, which is a conventional gradient-based optimizer, provided with ADAMS/Insight. During the optimization process, the design variables are adjusted so that the resulting responses (the root mean squares during simulation of the camber and castor variation) come as closely as possible to the specified target values (in this case zero).

The results shown in Figures 6 and 7 present the angle variations of the interest parameters in the initial and final tuning steps. There can be observed substantial reductions for both the design objectives, with the following root mean squares (RMS): camber angle: initial - 0.13123, final - 0.096515; castor angle: initial - 0.17596, final - 0.00035205.

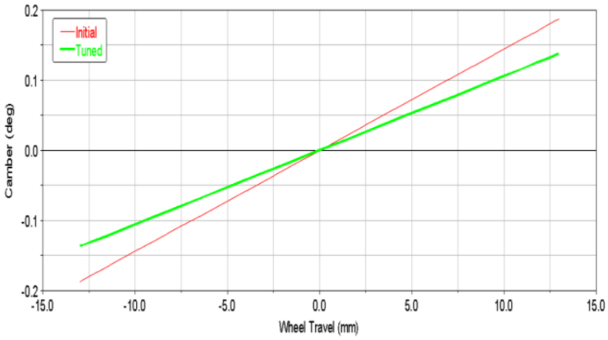

Figure 6: Camber angle variation (1st step)

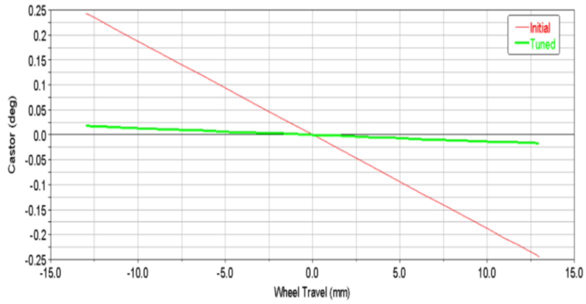

Figure 7: Castor angle variation (1st step)

Based on Figure 6, the initial camber angle variation (min -0.1867°, max 0.1875°, with a slope of 0.0144) was reduced to a min of -0.1364° and a max of 0.1377°, this is shown as well, by the wheel travel-camber angle slope reduction, from an initial value of 0.0144 to a tuned value of 0.0106. Figure 7 shows an initial castor angle variation of min -0.244° to a max of 0.2431° with a slope of 0.0188, that was tuned to achieve min -0.017°, max 0.0175°, with a slope of 0.0014. All these variations are in the acceptable domains for a double wishbone suspension, which demonstrates the viability of the adopted optimization strategy.

The steer angle variation during travel (i.e. bump steer) was not tuned at this step. The results in Figure 8 shows the influence of the camber & castor tuning on the bump steer angle variation, this started at: min -0.024°, max: 0.0148°, with a slope of 0.0013, and degraded to min -0.044°, max 0.0415°, with a slope of 0.0032. The first step tuning degradation of the bump steer was expected, this will be dealt with in the second step tuning and will be based on the inner and outer track rod link, and should not affect on the camber and castor tuning.

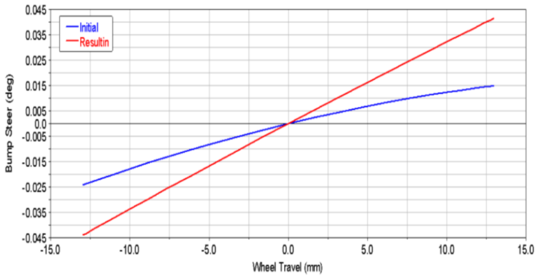

Figure 8: Bump steer variation (1st step)

The second step of the tuning process starts at the end of the first step, so the model will use the set of hard points, already tuned in step one, for the upper and lower control arms (DV_1 – DV_12).

The bump steer tuning process will be similar with the camber/castor tuning process, this will be using the track rod inner (to the chassis) and outer (to the wheel carrier) hard points. The Cartesian coordinates (x, y, z) of this hard points (see Fig. 2.) will be defined as the design variables, and will be able to move +/-40mm from their initial values. The design objective will be de reduction of the steering angle during wheel travel (bump steer) without affecting the camber and castor angles.

By using the same design specification as in the camber/castor tuning step, the DOE Screening (2 Level) (investigation strategy) with the interaction model and D-Optimal (DOE Design Type), and considering the minimum number of runs for the selected investigation strategy (22 trials) (Fig. 9), the resulting design space will be a matrix of 6 design variables over the 22 runs (shown in Table 3).

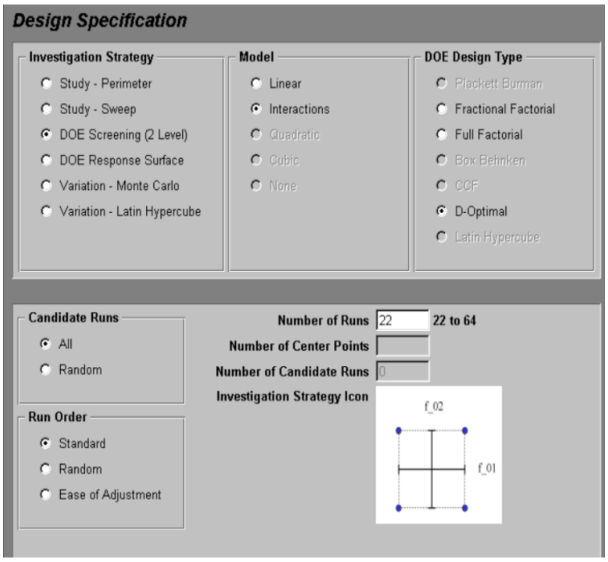

Figure 9: Design specifications for bump steer tuning

For each trial, a simulation will be performed, ADAMS/View completes the runs, and the results appear in the work space. For example, part from the work space is shown in Table 4 (trials no 1, 2, 3, 20, 21 and 22). As in the previous tuning step, in case of the goodness of fit and the fit for regression, the green indicator, show that the regression model is a good fit and does not need any changes (Figure 10 a,b).

Table 3. Design space for bump steer optimization

Trial	DV_19	DV_20	DV_21	DV_28	DV_29	DV_30
Trial 1	1	1	-1	-1	-1	-1
Trial 2	-1	1	1	-1	-1	-1
Trial 3	-1	1	-1	-1	-1	1
...	...	...	...	...	...	...
Trial 20	1	1	1	1	1	-1
Trial 21	1	1	1	-1	1	1
Trial 22	-1	-1	-1	-1	-1	-1

Table 4. Several trials from the work space of the bump steer experiment

Trial	DV_19	DV_20	DV_21	DV_28	DV_29	DV_30	B_S
Trial 1	290	945	260	80	200	280	*0.00294*
Trial 2	210	945	340	80	200	280	*0.18789*
Trial 3	210	945	260	80	200	360	*0.25156*
...	...	...	...	...	...	...	...
Trial 20	290	945	340	160	280	280	*0.14858*
Trial 21	290	945	340	160	200	360	*0.03467*
Trial 22	210	865	260	80	200	280	*0.01642*

Goodness-of-fit for model

	B_S
R2	1
R2adj	1
P	0
R/V	1e+20

a.

Fit for regression "B_S"

	DOF	SS	MS	F	P
Model	21	0.222	0.0106	1e+20	0
Error	0	3.91e-28	0		
Total	21	0.222			
R2	1				
R2adj	1				
R/V	1e+20				

b.

Figure 10: Goodness-of-fit summary (a) and fit regression (b) for the bump steer response

The tuning result are presented in Figure 11, this shows the bump steer angle variation. There is significant reductions for the bump steer parameter, with the following root mean squares (RMS): initial: 0.1232 final: 0.0321. The bump steer variations show a change from a initial min value of -0.044°, to max of 0.0415° with a slope of 0.0032° to a tuned of min -0.02°, max 0.0165° with a slope of 0.0012°.

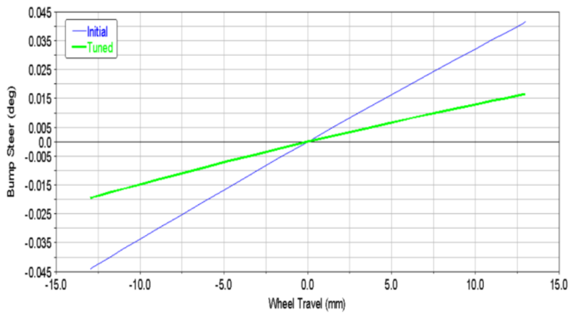

Figure 11: Bump steer variation (2nd step)

After the second step of the tuning process, the camber angle variation (Figure 12) was not affected, (initial slope is equal to the resulting slope: 0.0107). The castor angle variation (Figure 13) has a 7% change from a tuned slope of 0.0014 to 0.0013; the change is in the tuning direction (i.e. it reduces the angle variation during the wheel travel), so it is beneficial.

As the design objective was to minimize all the angle variations during travel, and all the above curves satisfy this objective, we can conclude that the proposed optimization steps process can be used to tune a double wishbone suspension system, no matter what the design objective will be.

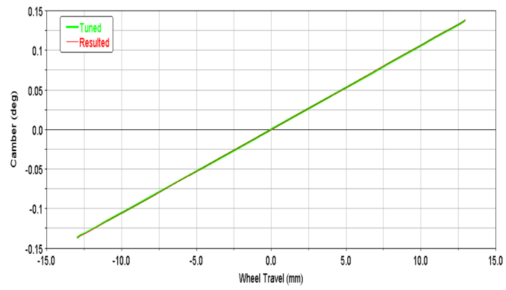

Figure 12: Camber angle variation (2nd step)

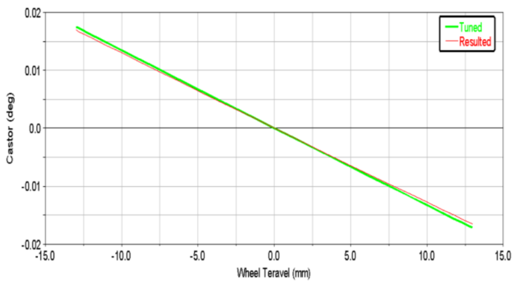

Figure 13: Castor angle variation (2nd step)

3. Final Results Check

The Static and Kinematic Suspension Parameter (S&KSP) software is an in-house made app, which was created by using the MATLAB programing software [8, 11, 16]. It computes the kinematic movement of the suspension components, for several front and rear suspension systems, based on a designed mathematical template. For the current paper, the front double wishbone layout was used.

For computing the double wishbone kinematic movement, the S&KSP software uses the rotation of a point around a axes in space mathematical calculation as follows: It takes into account the position of the upper control arm to the chassis hard points (Pt1 and Pt2) and rotates the arm to wheel carrier point (Pt5) in angle increments around the Pt1-Pt2 axis until Pt5 passes through the full inputted wheel travel (in 0.5 mm increments) and will save the x,y,z results (Figure 14).

	A	B	C	D
1		X	Y	Z
2	Pt_1	-430	80	490
3	Pt_2	547.44	130	465.4
4	Pt_3	-515.14	50	160
5	Pt_4	569.98	97.31	119.51
6	Pt_5	0	900	500
7	Pt_6	10	910	100
8	Pt_7	290	945	260
11	Pt_9	1	1200	131
12	Pt_10	10	1800	130
15	Pt_13	280	160	280
25	W_T		26	

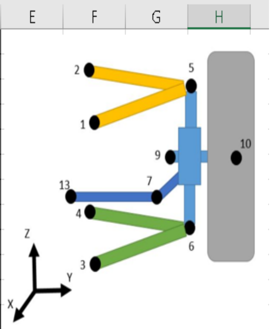

Figure 14: Double wishbone template

For example, the x,y,z value of the interest point will be computed in the following way:

$$x = \{[Pt1\_x \cdot (v^2 + w^2) - u \cdot (Pt1\_y \cdot v + Pt1\_z \cdot w - u \cdot Pt5\_x - v \cdot Pt5\_y - w \cdot Pt5\_z)] \cdot \{1 - \cos(A)\} + L \cdot Pt5\_x \cdot \cos(A) + \sqrt{L} \cdot (-Pt1\_z \cdot v + Pt1\_y \cdot w - w \cdot Pt5\_y + v \cdot Pt5\_z)\} \cdot \frac{\sin(A)}{L} \quad (2)$$

$$y = \{[Pt1\_y \cdot (u^2 + w^2) - v \cdot (Pt1\_x \cdot u + Pt1\_z \cdot w - u \cdot Pt5\_x - v \cdot Pt5\_y - w \cdot Pt5\_z)] \cdot \{1 - \cos(A)\} + L \cdot Pt5\_y \cdot \cos(A) + \sqrt{L} \cdot (Pt1\_z \cdot u + Pt1\_x \cdot w - w \cdot Pt5\_x + u \cdot Pt5\_z)\} \cdot \frac{\sin(A)}{L} \quad (3)$$

$$z = \{[Pt1\_z \cdot (u^2 + v^2) - w \cdot (Pt1\_x \cdot u + Pt1\_y \cdot v - u \cdot Pt5\_x - v \cdot Pt5\_y - w \cdot Pt5\_z)] \cdot \{1 - \cos(A)\} + L \cdot Pt5\_z \cdot \cos(A) + \sqrt{L} \cdot (Pt1\_y \cdot u + Pt1\_x \cdot v - v \cdot Pt5\_x + u \cdot Pt5\_y)\} \cdot \frac{\sin(A)}{L} \quad (4)$$

where A is the angle increment, u is the difference between the two chassis hard points on the x axis, v is the difference between the two chassis hard points on the y axis, w is the difference between the two chassis hard points on the z axis, and L is the square sum of u, v and w.

To calculate the lower ball joint movement, the software will use the three sphere intersection method. This method is based on defining three distances as being the three spheres radius and will calculate the two intersecting points.

The three sphere intersection method results in a number of two intersecting points in space, from which only in one set of the results will show the initial lower ball joint position, this is the set of results that will be used.

For example, the lower ball joint position will be calculated based on the following formula, where Pt6_1 and Pt6_2 are the two set of results found for the Pt 6 [x,y,z] coordinates:

$$Pt6_1 = p1 + u1 + \frac{\sqrt{r1^2 - \sum u1.^2} * c}{\sqrt{c2}}, \quad (5)$$

$$Pt6_2 = p1 + u1 - \frac{\sqrt{r1^2 - \sum u1.^2} * c}{\sqrt{c2}}, \quad (6)$$

in which

$$u1 = \frac{\left\{\frac{[(cross\ \Sigma(p21.^2) + r1^2 - r2^2) * p31 - (\Sigma(p31.^2) + r1^2 - r3^2) - p21]}{[2, c]}\right\}}{c2} \quad (7)$$

where p21 is the difference between the x,y,z point position of pt5(current position) and Pt3(lower control arm to chassis point) position, p31 is the difference between the position of Pt4 and Pt5(current position, c is the cross between p21 and p31, c is c2 squared, p1 is pt5 current position, p2 is the Pt3 position, p3 is Pt4 position, r1 is the distance between the initial Pt5 and Pt6, r2 is the distance between Pt6 and Pt3 and r3 is the distance between Pt6 and Pt4 (all these points are based on the software hard points definition shown in Figure 14).

The S&KSP software uses the three sphere intersection method to compute the position of the track rod to wheel carrier hard point (Pt7), and based on that position it will calculate the x,y,z kinematic coordinates of the bearing center (Pt9 - bearing center) and wheel center (Pt10 - wheel center). The app will compute the bump steer angle based on the wheel center and the bearing center points, with the following formula:

$$BS = tan^{-1}\left\{\frac{\left[\sqrt{\frac{(Pt9\_Y - Pt10\_Y)}{(Pt9\_X - Pt10\_X)}}^2 - \sqrt{\frac{(Pt9\_y - Pt10\_y)}{(Pt9\_x - Pt10\_x)}}^2\right]}{1 + \sqrt{\frac{(Pt9\_Y - Pt10\_Y)}{(Pt9\_X - Pt10\_X)}}^2 * \sqrt{\frac{(Pt9\_y - Pt10\_y)}{(Pt9\_x - Pt10\_x)}}^2}\right\} \quad (8)$$

where Pt9_X, Pt10_X, Pt9_Y, Pt10_Y are current x and y position of point 9 and 10, and Pt9_x, Pt10_x, Pt9_y, Pt10_y are the initial position of the interest points.

The camber and castor variations are computed in a similar way as the bump steer, but the inputs are Pt5 and Pt6 x,z coordinate for the castor angle variation and Pt5 and Pt6 y,z coordinate for the camber angle variation. All the above formulas are used in steps of 0.5mm wheel travel, the results are saved as angle variations and transformed from radians to degrees for a better understanding of the results and easier correlation.

The input in the S&KSP program will be the resulting hard points from the previous tuning steps. Although the S&KSP program gives more static and kinematic suspension parameters, in what follows only the camber, castor and bump steer variations (that is, those variations for which the optimization was performed in the 2nd section of the work) are considered. The graphical results of the S&KSP software are shown in Figure 15. In addition, Table 6 shows some relevant values for these variations, considering the wheel travel in the field +/-13mm.

There are several ways to correlate the MSC. ADAMS with the S&KSP results: by overlapping the graphical results, by comparing the min and max angles at the maximum wheel travel and by comparing the slope.

By comparing the graphical between MSC.ADAMS and the S&KSP software, we can see that the two software show similar linear results. But the two results are on two different graphs so it is hard to make shore all the graphs overlap 100%. The second correlation exercise is based on the min and max angles of the results. This is done by comparing the numerical results depicted in the 2nd section of the work with those in Table 6. As all the min and max angles are identical, we can safely say that the two models validate each other.

The third way to interrogate the results is by comparing the slopes. As long as the wheel travel is known, for a given slope the min and max angles can be back calculated, this seems to be best and safest way, as long as the "on center" results are linear.

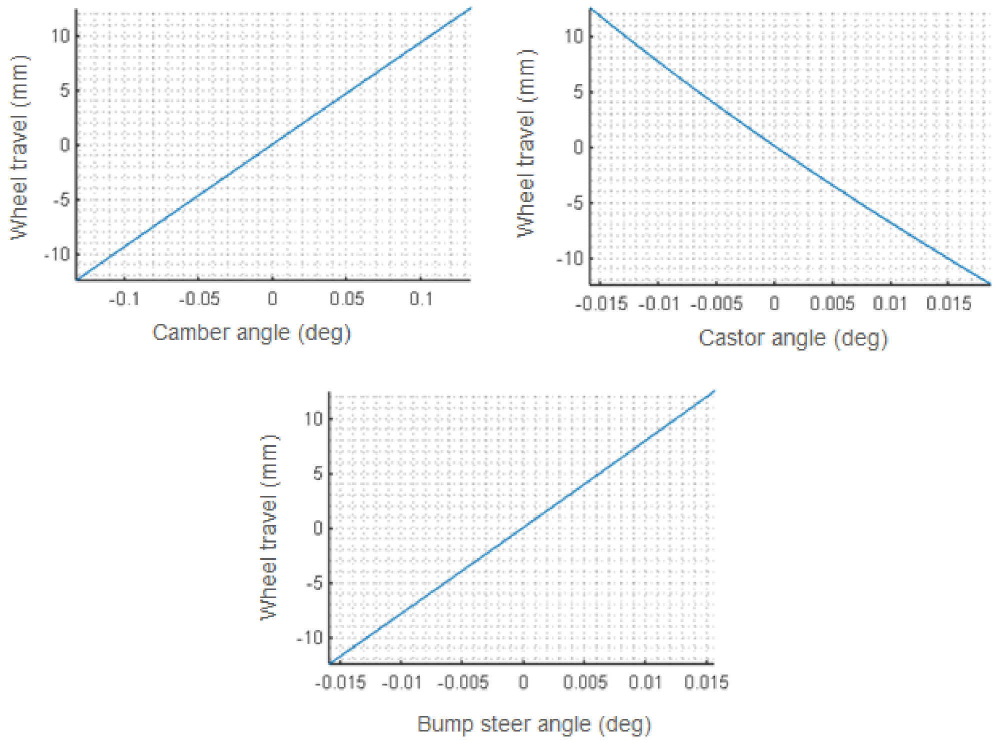

Figure 15: Graphical results from S&KSP

Table 6. Numerical values provided by S&KSP

Angle [°]	min	max	slope
camber	-0.136	0.138	0.011
castor	-0.017	0.018	0.0013
bump steer	-0.02	0.017	0.0012

4. Conclusions

The proposed optimization strategy leads to an efficient suspension, without developing expensive hardware prototypes. Thus, the behavioral performance predictions are obtained much earlier in the design cycle, thereby allowing more effective and cost efficient design changes.

The upper and lower control arms of a double wishbone suspension system have the most influence on the camber and castor angles. The bump steer angle is mostly influenced by the steering rack (track rod) position.

By running the models developed with the two software solutions (MSC.ADAMS and S&KSP), similar results were obtained, which leads to the mutual validation of the models. The best way to compare "on center" kinematic results is to compare the slope of the target curves as long as this are linear, not the min and max values of the design objectives, as the initial static values, are not given by the suspension hard points that the tuning will influence.

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