TECHNOLOGICAL SUPPORT FOR MANUFACTURING COMPLEX GEOMETRY HYPERBOLOID GEARS IN A CNC ENVIRONMENT

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Abstract - The development of modern engineering imposes ever-increasing requirements on gearings. Existing types of crossed-axis gears include worm, spiroid, hypoid, helical (crossed helical), and cylindrical-bevel gears. The recommended range of transmission ratios for worm and spiroid gears is between 20 and 60. A decrease in the gear ratio leads to a sharp increase in the risk of gear tooth undercutting and pointing, as well as worm thread undercutting and pointing; it also results in a reduced contact ratio. In the transmission ratio range of 1 to 4, helical and cylindrical-bevel gears may be used; however, they are unsuitable for heavy-duty drives. In mechanisms requiring high torque transmission at gear ratios below 10, hypoid gears are widely used as power transmissions. In heavy-duty crossed-axis drives, the contact pressure on the tooth flanks reaches at least 1500–2000 MPa, and in some cases up to 4000 MPa. This leads to lubricant film breakdown, metal-to-metal contact of the rubbing surfaces, and enormous shear stresses exceeding the limit of plastic deformation of the teeth. Theoretical studies of gears based on a single-sheet hyperboloid of revolution have shown that the theoretical contact ratio of a hyperboloid gear with double-curvature teeth is significantly higher than that of any currently used gear system in the 1–6 transmission ratio range. Manufacturing challenges for hyperboloid gears with double-curvature teeth can only be overcome through multi-axis machining. Efficient forming of such teeth is possible only by controlling the tool orientation.

Keywords: CNC, Hyperboloid Gears, Milling, Surface Generation, Gear Cutting, Five-Axis Machining.

1. Introduction

Technologically, all methods of generating gear tooth flanks through machining are traditionally categorized into three primary methods: copying (forming), generating (generating by rolling), and enveloping, or their combinations.

An involute profile can be produced using either the copying or the generating method. The copying method is implemented on specialized machines, universal milling machines equipped with indexing heads, or CNC machines. The disadvantages of this method include lower manufacturing precision compared to the generating method, typically achieving only accuracy grades 9–10. The advantages of the copying method include low tool costs (in mass production) and the ability to form diverse tooth shapes. In mass production scenarios using specialized machinery, such as gear broaching, this method often proves to be 10–20% more productive than the generating method [1-3]. When using the generating method for milling cylindrical gears, the tooth profile is formed by a sequence of changing positions of the cutting edge traces during their mutual rolling motion. In the enveloping method, the gear tooth surface is formed as an envelope of successive cutting edge positions in the absence of machine gearing between the workpiece and a virtual tool.

In addition to these three basic methods, gear-cutting techniques that combine these approaches, such as circular broaching, are used in practice. In this process, each generating line of the tool removes a chip, leaving a cutting edge trace in the form of a circular arc (copying method), while the tooth shape along its length is formed by the enveloping of these cutting edge traces in space (enveloping method).

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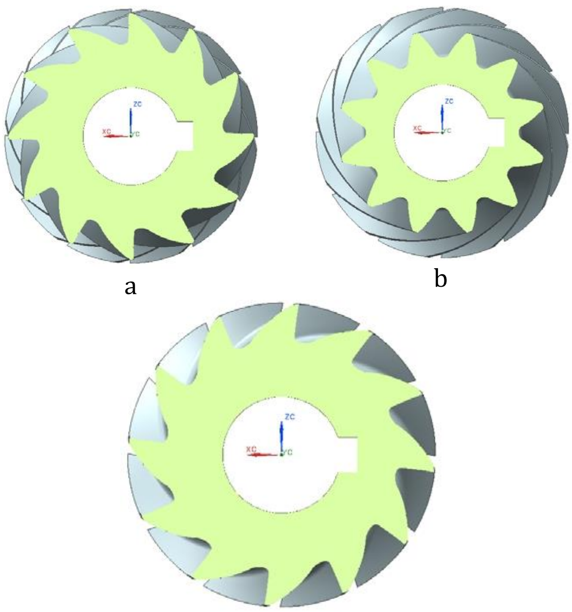
When the required forming motions can be achieved on conventional gear-cutting equipment, such machining is often the most optimal and economically justified solution.

However, the development of multi-axis CNC machines has made it possible to overcome several technological limitations inherent in classical gear cutting, facilitating the development of new tooth-forming methods. A hyperboloid gear with double-variable curvature teeth is defined as a gear system where the teeth are formed on a workpiece shaped as a single-sheet hyperboloid of revolution, with the tooth flanks changing their curvature both along the tooth height and its length.

2. Literature Review and Problem Statement

The manufacturing of double-curvature hyperboloid gears is a complex task. The complexity of producing the teeth for these gears arises from the fact that, in addition to the hyperboloid surface varying across the face width, both the tooth space and the tooth thickness must also be variable. Furthermore, the helix angle changes along the tooth length, as does its geometric position relative to the tooth height [4-7]. The variable curvature of the tooth flank (Figure 1) creates technical difficulties when using the generating method for cutting such gears.



a b c

Figure 1: Z-sections of a single gear tooth (the z-axis is oriented in the direction of view).
a) z=-13mm b) z=0 c) z=13mm

The variable tooth curvature and the varying tooth space width along the length of gears cut on single-sheet hyperboloid of revolution blanks prevent the use of high-performance generating methods, as well as other methods used for forming teeth in helical gears.

High requirements are placed on the surface finish quality of the tooth flanks, which are difficult to achieve during gear-cutting operations; therefore, the manufacturing process for these gears includes gear-finishing operations.

It is well known that two types of machining are used for complex-profile surfaces: those with a free kinematic link between the workpiece and the tool, and those with a fixed kinematic link[8-11]. Forming with a fixed kinematic link in the 'tool-workpiece' system allows for the automatic achievement of the required dimensional accuracy and geometric shape. So-called machining with a free kinematic link ensures the necessary surface roughness and physical-mechanical properties of the surface layer.

Dimensional machining with bladed tools is performed using hobs, gear shapers, rack-type cutters, shavers, and generating cutters with an accuracy up to grade 6 and roughness up to Ra = 0.63 μm. Due to the variable transverse module, variable tooth pitch, and specific tooth shape, the aforementioned methods are unsuitable for the double-curvature hyperboloid gears discussed in this study.

Shaving allows for a reduction in profile error, pitch direction, and pitch circle runout, while decreasing surface roughness to Ra = 0.32 μm. Gear shaving is one of the most productive and cost-effective finishing methods. The disadvantages of this method include the impossibility of machining hardened gears, as well as the technological complexities of manufacturing such tools. Gear honing is widely used for tooth processing; unlike lapping, it does not cause charging (embedding of abrasive particles) of the working surface. It provides significant material removal compared to generating, polishing, and superfinishing methods, and can even be more productive than grinding. For hypoid and bevel gears, this method is implemented on machines with an angular configuration[12-18]. The tool used is a specially designed metal gear coated with a layer of cubic boron nitride (CBN). This method increases gear durability by 2–3 times.

The purpose of gear drives is to transmit rotation. Gears have become widely used due to their high reliability and design features that ensure compact drive dimensions with high efficiency, reaching up to 98%. The most critical elements of gears that facilitate rotation transmission are the tooth profiles. The technological advantages of involute gearing, specifically a two- to three-fold reduction in the variety of required cutting tools, have made this profile shape the industry standard. In theoretically precise drives with parallel axes, pure rolling occurs at the pitch point as the tooth engages; consequently, there is almost no wear at this location. However, maximum sliding occurs at the tip and root of the tooth, resulting in significantly higher wear in those areas.

Surface formation by cutting is widely used across all industries. Complex surface profiles can be produced using various manufacturing methods. Due to the technical limitations of traditional gear-hobbing machines in executing more than three simultaneous shaping motions, complex-profile surfaces produced on such equipment were often replaced with simpler geometries, introducing additional formation errors. Manual programming or on-machine programming for complex-contoured parts is impractical due to high labor intensity and the complexity of calculating toolpath trajectories. Furthermore, entering control programs directly at the machine leads to additional downtime. These drawbacks are addressed by programming through Computer-Aided Manufacturing (CAM) systems. The maximum efficiency of CNC machines is achieved when forming complex-profile surfaces, which is often an economically viable solution in single-unit and small-batch production. Preparing a control program using CAM systems requires geometric modeling of both the blank and the finished part [19-20]. Therefore, surface and solid modeling of the designed product is generally the central task of design and engineering support for CNC machining. A shift in traditional approaches to surface formation has led to a new differential-geometric method, where the target part surface is primary, while the methods and means of processing (including the cutting tool) are considered secondary.

3. Methodology, Results and Discussion

The analytical derivation of dependencies for calculating the generatrix during the forming of the tooth flank will be performed for a specific case of forming the flanks of a hyperboloid gear. The calculation is carried out for a gear with a unit gear ratio and an axis crossing angle of 90 degrees (Figure 2). Mathematical dependencies for calculating the coordinates of the generatrix points of the conjugate generating surfaces will be obtained in parametric form, assuming a line segment is chosen as the generatrix.

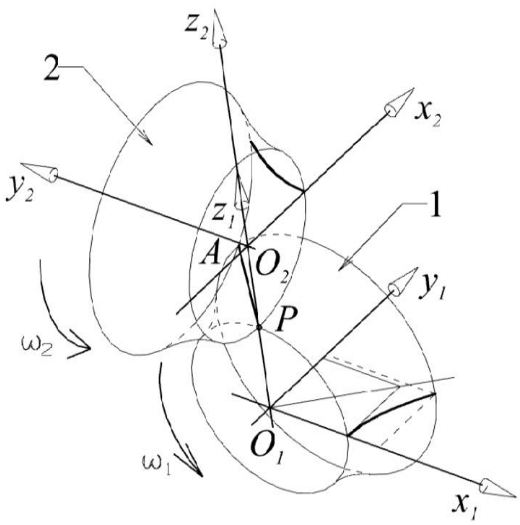


Figure 2: Reference hyperboloids of the gear drive:1 — reference hyperboloid of the gear being machined;2 — reference hyperboloid associated with the generatrix; ω_1, ω_2 — direction of rotation of the reference hyperboloids during the surface forming process.

Since the reference hyperboloids are solids of revolution, the calculation of the generatrix will be performed for the spatial translation of the generatrix relative to the hyperboloids involved in the gear drive. Investigating the motion of a point on the generatrix during the forming of the tooth flank will allow for obtaining the tooth lead (guide line). The conjugate surfaces for each hyperboloid are calculated separately. The calculation will be carried out only for gear 1, as the derivations for gear 2 will be identical. We attach fixed coordinate systems and to the hyperboloids $O_1X_1Y_1Z_1$, $O_2X_2Y_2Z_2$ (Figure 2). The hyperboloids are surfaces of revolution of hyperbolas, the equations of which in parametric form are as follows:

$$x_i = a \cdot ch(t) \tag{1}$$

$$y_i = b \cdot sh(t) \tag{2}$$

where a – is the real semi-axis and b – is the imaginary semi-axis of the hyperbola; i – is the index of the hyperboloid and the corresponding coordinate system; *sh(t)* and *ch(t)* – are the hyperbolic sine and cosine, respectively.

Hyperboloid 1 represents the reference hyperboloid of the gear being machined. Hyperboloid 2 reproduces the motion of the gear mating with gear 1. The generatrix, which forms the tooth flank profile on hyperboloid 1, is associated with gear 1.

Let us consider the motion of the generatrix in the coordinate system $O_2X_2Y_2Z_2$ (Figure 3). Since the generatrix is a line segment, it is sufficient to consider the motion of two of its points: M_1 and M_2. We take M_1 and M_2 as the endpoints of the generatrix. By investigating the motion of these endpoints during the forming of the tooth flank, we can determine the tooth lead (guide line).

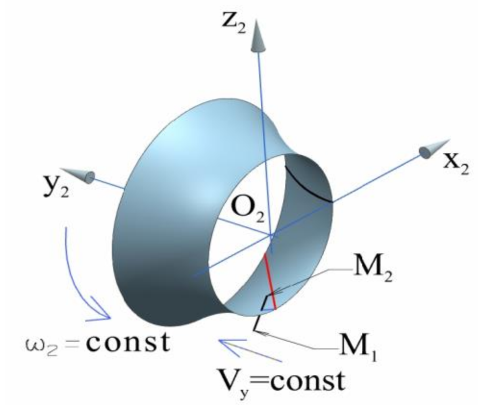


Figure 3: Diagram for calculating the generatrix motion during the forming of the tooth flank: M_1 is the point of the generatrix forming the tooth root; M_2 is the point of the generatrix forming the tooth addendum; Vy is the translation velocity of points M_1 and M_2 along the y_2-axis.

When transmitting rotation between gears with crossed axes, the relative motion of the links, as is well known from theoretical mechanics, is helical (screw motion). The velocity of the relative motion, represented as helical motion, is equal to the sum of the velocities of the rotational motion around the screw axis and the translational motion along the same axis. Each point belonging to the generatrix participates in two motions: rotation around the y_2-axis with a constant angular velocity ω_2 (transport motion) and motion along a hyperbola along y_2 with a constant velocity projection onto the y_2-axis (relative motion).

The initial positions of points M_1 and M_2 are defined in a section normal to the direction of the tooth flank and passing through the common perpendicular.

The endpoints of the generatrix, M_1 and M_2, correspond respectively to the regions of the hyperboloid (the lower part, or dedendum) of gear 1 (point M_1) and the tooth tip (addendum) region of gear 1 (point M_2). Each point moves in relative motion within the $X_2O_2Z_2$ coordinate system along its own hyperbola; we shall limit our consideration to point M_1.

$$xM_1 = a_{M1} \cdot ch(t) \tag{3}$$

$$yM_1 = b_{M1} \cdot sh(t) \tag{4}$$

where a_{M1} is the real semi-axis and b_{M2} is the imaginary semi-axis of the hyperbola passing through point. For proper tooth engagement, the necessary conditions are a constant tangential velocity and a constant motion of the point relative to the y_2-axis. Therefore, we obtain the following relationship:

$$d\left(y_{M1}\right)/dt = d\left(b_{M1} \cdot sh(t)\right)/dt = const \tag{5}$$

In the transport motion, the point rotates together with the hyperboloid associated with the generatrix. The angular position of the generatrix relative to the x_2-axis, in a plane parallel to the $X_2O_2Z_2$ plane, is defined by the following equation:

$$\varphi_{M1} = \varphi_{M1_0} + \omega_2 t \tag{6}$$

where φ_{M10} is the initial angular position of point M_1.

Hence, to calculate the coordinates of point M_1 undergoing combined motion—i.e., to define its coordinates parametrically within the coordinate system linked to the generatrix—we obtain the following relationships:

$$\begin{pmatrix} x_2(t) \\ y_2(t) \\ z_2(t) \\ t \end{pmatrix} = \begin{pmatrix} a_{M1} ch(t)\cos(\varphi_{M0} + \omega t) \\ h_2 b_{M1} sh(t) \\ a_{M1} ch(t)\sin(\varphi_{M0} + \omega t) \\ t \end{pmatrix} \tag{7}$$

where h_2 is the helical pitch coefficient, calculated as follows:

$$h = \frac{u^*}{\omega_i} \tag{8}$$

where ω is the angular velocity of the helical motion, u^* is the translational velocity of the helical motion.

With a gear ratio u=1 and an axis crossing angle $\sum 90^0$, the helical pitch coefficient h=1.

By varying the parameter t, matrix (7) allows for calculating the coordinates of point M_1in the $O_2X_2Y_2Z_2$ coordinate system. For this purpose, the initial position of point M_1, the range of t, and the calculation step were defined to ensure the required accuracy.

To obtain the coordinates of the generatrix involved in the shaping of the tooth flank of the gear 1 (Figure 2), it is considered that the generatrix associated with gear 2 rotates relative to gear 1 at a constant angular velocity ω_1. The points of the lead line, formed on the tooth surface of the gear being machined by the motion of point M_1 as parameter t changes, will have coordinates calculated using the transition dependencies from the $O_2X_2Y_2Z_2$ coordinate system to the $O_1X_1Y_1Z_1$ coordinate system. The coordinates of point M_1 for the transition from $O_2X_2Y_2Z_2$ to $O_1X_1Y_1Z_1$ for the case under consideration

(Figure 3) are determined by the following relationships:

$$x_1(t) = O_1O_2 \sin(\omega t) - y_2 \cos(\omega t) + z_2 \sin(\omega t);$$
$$y_1(t) = x_2(t) = a_{M1} ch(t) \cos(\varphi_{M0} + \omega t); \quad (9)$$
$$z_1(t) = O_1O_2 \cos(\omega t) + y_2 \sin(\omega t) + z_2 \cos(\omega t)$$

where O_1O_2 (Figure 2) is the line segment connecting the centers of the hyperboloids—their center distance; x_2, y_2, z_2 are the coordinates of point M_1 in the $O_2X_2Y_2Z_2$ coordinate system, calculated according to relationship (7).

To calculate the relationship between the angular velocity of the gear and the translational velocity of the projection of point M_1's velocity (of the generatrix) onto the rotation axis of gear 2 (Figure 2), let us consider Figure 4.

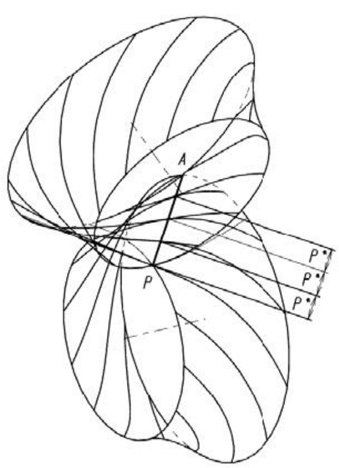


Figure 4: Calculation of helical lines along which the gear teeth are formed.

The gear has a specified number of teeth Z. Upon rotation through an angle of $360^0/Z$, the projection of point M_1 onto line PA must shift by the pitch value P^*. According to the theory of crossed-axis gearing, the pitch P^* must remain constant despite the hyperbolic increase in diameter; accordingly, the transverse module changes for a fixed number of teeth, which is accounted for in relationship (7). The trajectory of point M_2 of the generatrix M_1M_2 is calculated similarly to model the tooth flank of gear 1. Then, using the same approach, the conjugate surface for gear 2 is calculated, assuming gear 2 is stationary while the generating gear 1 rolls around it.

For a gear ratio of unity and an axis crossing angle of 90 degrees, we obtain the following equation:

$$\begin{pmatrix} x_1(t) \\ y_1(t) \\ z_1(t) \\ t \end{pmatrix} = \begin{pmatrix} O_1O_2 \sin(\omega t) - y_2 \cos(\omega t) + z_2 \sin(\omega t) \\ O_1O_2 \cos(\omega t) + y_2 \sin(\omega t) + z_2 \cos(\omega t) \end{pmatrix} \quad (10)$$

The derivation of mathematical dependencies for determining the end mill trajectory during five-axis machining will be performed for the gear (Figure 5) using the principle of generating engagement. The calculation will be carried out for the tool generatrix AB, a segment of which, during the motion reproducing the engagement, forms the tooth flank in the process of end milling.

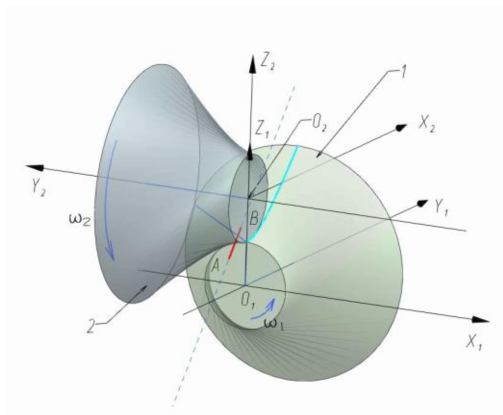


Figure 5: Calculation of the generatrix orientation.

The initial position of the generatrix AB is defined in a section normal to the direction of the tooth flank and passing through the common perpendicular.

Two hyperboloid gears, represented by their reference hyperboloids, are in mesh and rotate with angular velocities ω and about the Y_1 and Y_2 axes, respectively. Let us consider the kinematics of surface forming in relative motion. Assume that gear 1 and the $X_1Y_1Z_1$ coordinate system are stationary. The $X_2Y_2Z_2$ coordinate system and the associated gear 2 rotate about the Y_1 axis with a constant angular velocity ω, but in the opposite direction. The Y_2 axis is the axis of rotation for hyperboloid 2, around which it rotates with an angular velocity ω. The generatrix line, containing the segment AB, rotates together with gear 2 about the Y_2 axis and simultaneously moves translationally along the Y_2 axis. The surface described by the generatrix relative to gear 1, within the range between the nominal surfaces corresponding to the tooth tips and roots, will correspond to the tooth flank of the gear.

The tool tracing point trajectory and the tool orientation must be determined at each point of the trajectory (Figure 6).

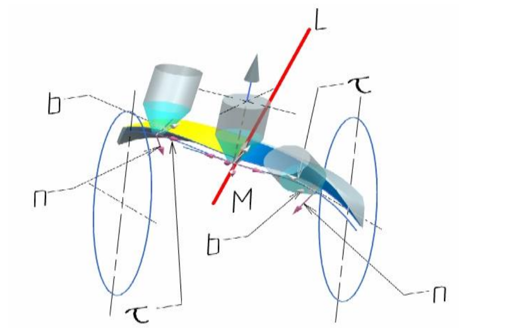


Figure 6: Calculation of the tool trajectory and orientation: LM — generatrix line; τ — unit tangent vector; n — unit normal vector; b — unit binormal vector.

The equations of motion of the line form a ruled surface:

$$\frac{x_1(t)-x_1^A(t)}{l(t)}=\frac{y_1(t)-y_1^A(t)}{m(t)}=\frac{z_1(t)-y_1^A(t)}{n(t)} \qquad (11)$$

For t=0, the initial points of the generatrix forming the tooth flank of gear 1 are determined. The initial position of the points on the generatrix *AB* is defined in a section normal to the direction of the tooth flank and passing through the common perpendicular.

The remaining points are calculated by varying the parameter *t*. The calculation must be performed with the parameter changing in both positive and negative directions. From the entire calculated surface, the tooth flank will be formed by the segments bounded by the surfaces of revolution that form the tooth tips and roots.

The equation of the reference hyperboloid is as follows:

$$\frac{x_1^2}{a^2}+\frac{z_1^2}{a^2}-\frac{y_1^2}{b^2}=1 \qquad (12)$$

For the transverse tooth module in the throat section *mn*, the following relationship holds true:

$$m_n(0)=\frac{D(y=0)}{z_1^*}=\frac{2a}{z} \qquad (13)$$

where z_1 is the coordinate, z_1^* is the number of teeth of gear 1, a is the real semi-axis of the hyperbola, and b is the imaginary semi-axis of the hyperbola.

To find the lines of intersection between the surfaces bounding the tooth flank and the surface formed by the generatrix, we set equation (11) equal to the parameter t_1, which defines the orientation of the generatrix.

$$\frac{x_1(t)-x_1^B(t)}{l(t)}=\frac{y_1(t)-y_1^B(t)}{m(t)}=\frac{z_1(t)-y_1^B(t)}{n(t)}=t_1 \qquad (14)$$

where x_1^B, y_1^B are the coordinates of point *B*.

The coordinates of the tool orientation angle and the tracing point allow for the generation of a control program for a CNC machine, where it is additionally required to account for technological parameters, machining conditions, and maneuvering geometry. The calculated trajectory enables a more efficient utilization of the actual geometric properties of the tooth surface during machining, which significantly reduces processing time while achieving maximum accuracy and productivity compared to the toolsets for developing control programs from 3D models available in current CAM systems. For roughing stages, when removing bulk material, the same calculation results can be used by offsetting the tool in the direction of the vector by the amount of the machining allowance left for the subsequent stage.

If the length *BA* exceeds the length of the tool's cutting edge, the machining must be performed in several passes (step-over machining). In this case, for each subsequent pass, the tracing point is offset from the calculated one in the direction of vector BA by a value ensuring the removal of the specified layer; the mill then moves toward the calculated tracing line in the direction opposite to vector *BA*.

This mathematical framework allows for the direct programming of double-curvature hyperboloid gear machining using a transcoding program based specifically on the gear's geometric parameters. According to the derived dependencies, a macro-program can be developed for a CNC machine without the use of Computer-Aided Manufacturing (CAM) systems, taking into account gear parameters, tool geometry, and machining conditions.

A necessary constraint on the tool geometry is the verification that the tool does not interfere with the adjacent tooth while machining the flank of the current tooth.

4. Conclusions

In the course of the conducted research, mathematical dependencies for calculating the coordinates of the generatrix points were obtained. These allow for the creation of geometric models of tooth flanks with double variable curvature, the shape of which is generated by the motion of the generatrix along the lead line. A mathematical model for controlling the tool axis orientation angle on a five-axis CNC machine has been developed, enabling the machining of the tooth flank during the forming of double-variable-curvature hyperboloid gears. The resulting mathematical model allows for the machining of double-variable-curvature hyperboloid gear teeth through direct data input into the CNC machine without the use of CAM systems. Furthermore, an algorithm is presented for calculating and inputting the obtained orientation vector values and tool tracing point coordinates into software for control program preparation when implementing the developed tooth forming kinematics.

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