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IMPROVING THE HANDLING OF PASSENGER CARS THROUGH THE USE OF A COMBINED STEERING METHOD

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Abstract - The use of in-wheel motors (both electric and hydraulic) in vehicles equipped with combined power units makes it possible to enhance their handling and maneuverability. The handling performance of a vehicle during cornering can be improved by applying a combined steering method, which consists of simultaneously using the steerable wheels of the front axle and generating a torque difference between the driving wheels of the rear axle. This study investigates the possibility of utilizing the yaw moment generated by the difference in torque between the rear driving wheels to overcome the static resistance moment during turning, while the steering angles of the front wheels are used to establish the required kinematic parameters of the vehicle's turn. The goal of the work is to improve vehicle handling by eliminating the "dead-zone–type" nonlinearity in steering response. The objective is achieved by developing a mathematical model of the turning process, taking into account the distribution of forces and moments acting on the vehicle. The scientific novelty lies in establishing the conditions for rational torque distribution between the rear wheels, which ensures proportionality between steering input and vehicle response. The practical significance of the study is related to the possibility of applying the proposed approach in modern vehicles with distributed drive systems to improve stability, maneuverability, and safety.

Keywords: Combined steering, Vehicle handling, Torque vectoring, Yaw moment, In-wheel motors, Vehicle dynamics, Maneuverability, Steering control.

1. Introduction

By assessing the factors that cause environmental degradation through the emission of polluting gases, work is underway to introduce alternative energy sources for both specialised and transport vehicles. One of the ways in which the industry is developing is through the introduction of dual-flow transmission units, which is particularly noticeable in agricultural machinery.

Meeting speed and energy requirements while complying with operational standards is the primary objective of any modernization of agricultural equipment. Therefore, from the initial stage of designing the power plant (combustion engine and transmission), the conditions are created to increase the productivity and efficiency of agricultural equipment in a sustainable mode of operation. However, transient processes, i.e. acceleration and deceleration of vehicles, cannot be neglected, especially when studying a power plant with a dual-flow transmission.

The creation and effective use of modern tractors with ploughs is the basis for the development of the agro-industrial complex of any state. Today's farms use wheeled tractors equipped with conventional continuously variable transmissions (CVTs), in particular hydrovolumetric mechanical transmissions (HMCVT). The use of tractors with CVT is explained by undoubted advantages compared to transmissions: smooth running, improved ergonomics during technological work, automation of control, etc.

2. Purpose and Object of the Study

Maneuverability is one of the most important operational properties of automobiles and other wheeled vehicles.

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Therefore, a substantial number of scientific studies have been devoted to investigating the complex operational characteristic of vehicle maneuverability, as well as its simpler constituent properties—handling, steerability, and stability [1–6].

The combined steering method has found its primary application in wheeled tractors, and therefore numerous studies have focused on improving the maneuverability of wheeled tractors using this approach [1, 2]. At the same time, recent studies on electric vehicles with independently actuated wheels have demonstrated that torque vectoring and direct yaw-moment control can substantially improve vehicle response to steering-wheel inputs, yaw stability and overall handling performance [7–13]. These studies further show that the force-based control action can be coordinated with additional steering subsystems, including active front steering and rear-wheel steering [9, 12, 14], while modern steering-system architectures increasingly combine steering, braking, and torque-vectoring functions to improve controllability and functional redundancy [16]. The rapid development of in-wheel motor drive systems provides the technological basis for implementing such control strategies in modern vehicles [15].

In study [1], a dynamic model of the turning process of an all-wheel-drive wheeled vehicle using a combined steering method was examined (Fig. 1).

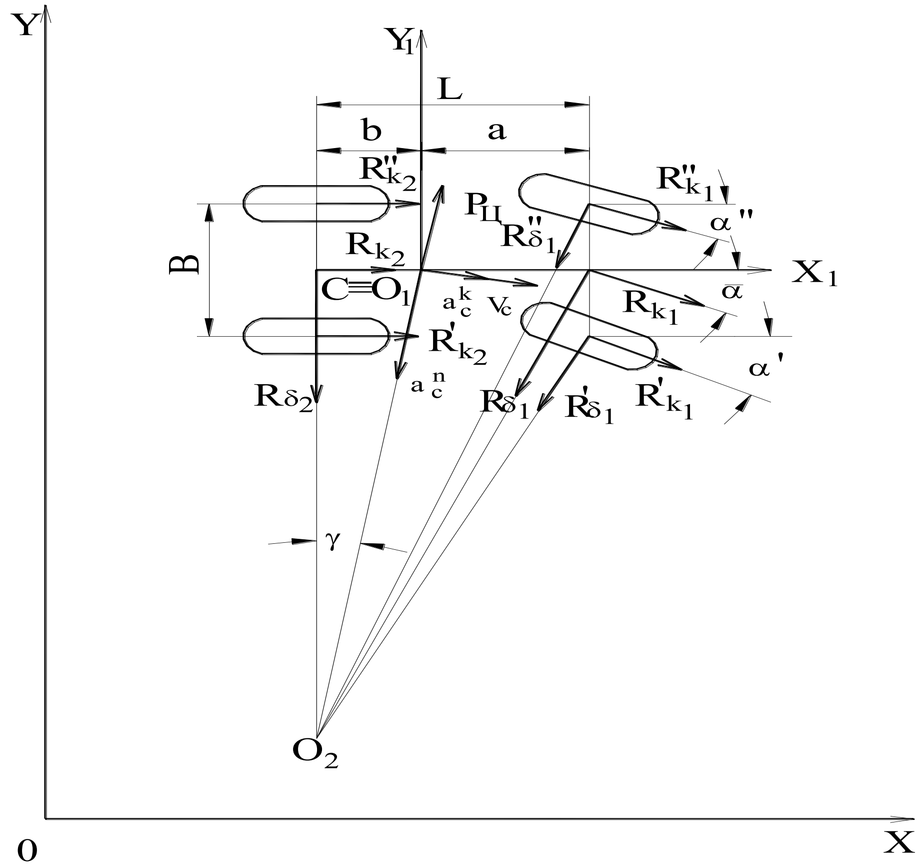

Figure 1: Scheme of the turning process of a four-wheel-drive vehicle with a front steering axle [1]

In study [1], the equation of the turning dynamics of a four-wheel-drive vehicle with a front steering axle was obtained, and it has the following form:

$$\frac{d}{dt}\left(J_{zO_2}\cdot\omega_z\right) = R''_{k_2}\left(R_2+\frac{B}{2}\right)+R'_{k_2}\times$$
$$\times\left(R_2-\frac{B}{2}\right)+R''_{k_1}\cdot R''_1+R'_{k_1}\cdot R'_1, \quad (1)$$

where J_{zO_2} – is the moment of inertia of the vehicle relative to the turning center O_2

$$J_{zO_2} = m_2\left(i_z^2+b^2+L^2\cdot ctg^2\bar{\alpha}\right), \quad (2)$$

ω_z - angular velocity of the vehicle in the plane of the road;

i_z - radius of gyration of the vehicle relative to the vertical axis;

L - longitudinal wheelbase of the vehicle;

b - distance from the rear axle to the projection of the vehicle's center of mass onto the horizontal plane;

m_a - vehicle mass;

B - vehicle track width (assumed identical for the front and rear axles);

R_2 - turning radius of the midpoint of the rear axle,

$$R_2 = L\cdot ctg\bar{\alpha}, \quad (3)$$

$\overline{\alpha}$ - mean steering angle of the vehicle's front steerable wheels;

$R_{K_1}^{'};R_{K_1}^{"}$ - tangential road reactions on the inner and outer front wheels of the vehicle;

$R_{K_2}^{'};R_{K_2}^{"}$ - tangential road reactions on the inner and outer rear driving wheels;

$R_1^{'};R_1^{"}$ - turning radii of the inner and outer front wheels.

In study [3], an analysis was carried out of the full range of indicators and criteria of maneuverability for wheeled vehicles. All of them were grouped into three main categories: force-related, kinematic, and energy-related. Under the combined steering method, two control actions are generated: a force action (yaw moment) and a kinematic action (steering of the guide wheels). This interpretation is consistent with modern integrated chassis-control concepts, in which steering determines the kinematic parameters of the maneuver, whereas torque vectoring or direct yaw-moment control supplies the corrective yaw action required for stability and handling improvement [9, 12–14].

During vehicle turning, the driver performs the function of the feedback element. However, the presence of a static moment of resistance to turning disrupts the proportionality between the steering input and the angular acceleration of the vehicle in the plane of the road.

The angular acceleration of a vehicle in the plane of the road is an indicator and criterion of its handling performance [1, 2, 3]. Studies [3, 4] have shown that the presence of a static moment of resistance to turning leads to the appearance of a "dry friction–type" nonlinearity, which degrades vehicle handling (see Fig. 2).

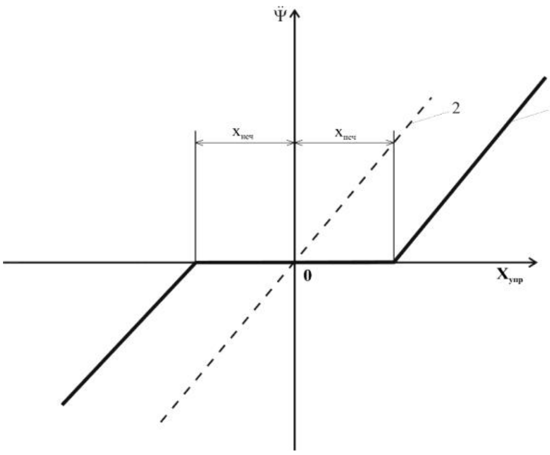

Figure 2: Dependence of the vehicle's angular acceleration in the road plane on the control effort: 1 — with a dead zone present; 2 — without a dead zone [3]

Vehicles equipped with in-wheel motors mounted on the rear driving wheels enable the implementation of a combined steering method, in which the difference in torque between the rear driving wheels generates a yaw moment that can be used to improve turning response and vehicle stability [7, 8, 10, 11, 13]. Owing to the high controllability of independently actuated wheels, in-wheel motor drive systems are especially suitable for the realization of such control strategies [15]. In this case, the vehicle's steering system exhibits a response without a dead zone, resulting in improved handling performance (Fig. 2, curve 2).

The objective of this study is to enhance the handling of vehicles with a rear driving axle equipped with in-wheel motors by utilizing a combined steering method during turning.

To achieve this objective, the following tasks must be addressed:

to determine the yaw moment generated by the torque difference between the rear wheels;

to establish the condition for rational control of the turning dynamics of front-wheel-drive vehicles;

to establish the condition for rational control of the turning dynamics of all-wheel-drive vehicles.

3. Determination of the Yaw Moment Generated by the Torque Difference Between the Rear Wheels of a Two-Axle Vehicle

To address the stated task, let us consider equation (1), which can be transformed into a form applicable to a rear-wheel-drive vehicle.

$$\frac{d}{dt}\left(Y_{zO_2}\cdot\omega_z\right)=R_{k_2}^{"}\left(R_2+\frac{B}{2}\right)+$$

$$+R_{k_2}^{'}\left(R_2-\frac{B}{2}\right)+R_{k_1}^{"}\cdot R_1^{"}+R_{k_1}^{'}\cdot R_1^{'}. \quad (4)$$

Let us adopt the following assumption (see Fig. 1):

$$R_{k_1}R_1=R_{k_1}^{'}\cdot R_1^{'}+R_{k_1}^{"}\cdot R_1^{"}, \quad (5)$$

where R_{K_1} - is the total tangential road reaction acting on the wheels of the front axle;

R_1 - is the turning radius of the midpoint of the vehicle's front axle,

$$R_1=\frac{R_2}{ctg\overline{\alpha}}=\frac{L}{sin\overline{\alpha}}. \quad (6)$$

The left-hand side of equation (4) can be expressed as follows:

$$\frac{d}{dt}\left(Y_{zO_2}\cdot\omega_z\right)=Y_{zO_2}\frac{d\omega_z}{dt}+\omega_z\frac{dY_{zO_2}}{dt} \quad (7)$$

By differentiating equation (2), we obtain:

$$\frac{dY_{zO_2}}{dt}=-2m_2L^2\frac{cos\alpha}{\sin^3\alpha}\cdot\frac{d\overline{\alpha}}{dt}. \quad (8)$$

In equation (4),

$$R'_{k_2} + R''_{k_2} = R_{k_2}\,, \tag{9}$$

where R_{K_2} - is the total tangential road reaction acting on the rear driving wheels;

$$M_{ПОВ} = \frac{B}{2}\left(R''_{k_2} - R'_{k_2}\right)\,. \tag{10}$$

The total tangential road reaction acting on the front steerable wheels

$$R_{k_1} = m_a \cdot g \cdot f\,\frac{b}{L}\,, \tag{11}$$

where f- is the rolling resistance coefficient of the wheels;

g - is the acceleration due to gravity, $g = 9{,}81\,м/c^2$

The tangential road reactions acting on the rear driving wheels

$$R'_{k_2} = \frac{M'_{k_2}}{r_\partial} - R'_{z_2} \cdot f\,; \tag{12}$$

$$R''_{k_2} = \frac{M''_{k_2}}{r_\partial} - R''_{z_2} \cdot f\,, \tag{13}$$

where $R'_{z_2}; R''_{z_2}$ - are the normal road reactions acting on the inner and outer rear driving wheels, respectively.

To determine the normal reactions $R'_{z_2}\, u\, R''_{z_2}$ let us consider the loading diagram of the vehicle during cornering (Fig. 3).

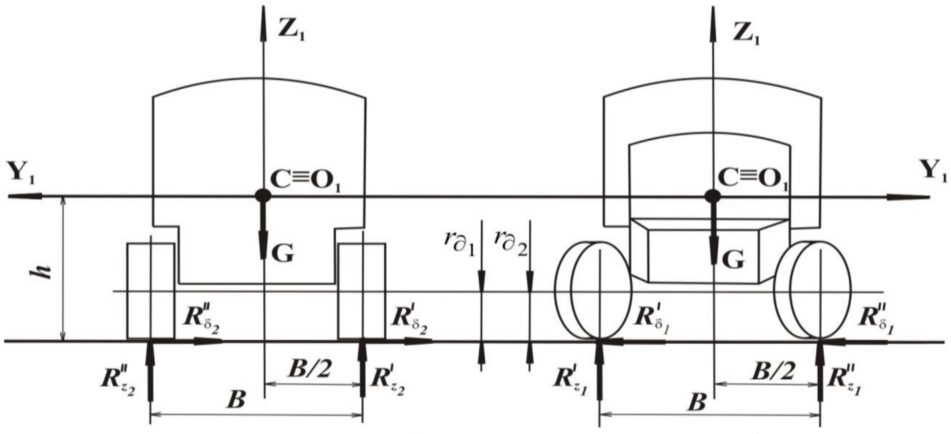

Figure 3: Diagram of the forces acting on a vehicle in the lateral plane during cornering:a – rear view;b – front view.

The normal road reactions on the rear wheels during vehicle cornering (Fig. 3a)

$$R'_{z_2} = 0.5 \cdot m_a \cdot g\,\frac{a}{L} - Pц\,\frac{a}{L} \cdot \frac{h}{B}\,; \tag{14}$$

$$R''_{k_2} = 0.5 m_a g\,\frac{a}{L} + Pц\,\frac{a}{L}\frac{h}{B}\,, \tag{15}$$

where a - is the distance from the front axle to the projection of the vehicle's center of mass onto the horizontal plane (see Fig. 1);

$P_ц$ - is the centrifugal inertial force,

$$Pц = m_a \cdot \omega_z^2 \cdot R_c\,; \tag{16}$$

R_C - the turning radius of the vehicle's center of mass,

$$R_c = \sqrt{R_2^2 + b^2} = \sqrt{L^2 ctg^2\bar{\alpha} + b^2} = L\sqrt{\frac{b^2}{L^2} + ctg^2\bar{\alpha}} \tag{17}$$

Equations (3), (14), and (15), taking into account relations (16) and (17), take the following form:

$$R'_{z_2} = 0.5 \cdot m_a \cdot g\,\frac{a}{L}\left(1 - 2\frac{\omega_z^2}{g} \cdot \frac{h}{B} L\sqrt{\frac{b^2}{L^2} + ctg^2\bar{\alpha}}\right); \tag{18}$$

$$R''_{z_2} = 0.5 \cdot m_a \cdot g\,\frac{a}{L}\left(1 + 2\frac{\omega_z^2}{g}\frac{h}{B} L\sqrt{\frac{b^2}{L^2} + ctg^2\bar{\alpha}}\right). \tag{19}$$

Expression (10), taking into account relations (12), (13), (18), and (19), takes the following form:

$$M_{ПОВ} = \frac{B}{2r_\partial}\left(M''_{k_2} - M'_{k_2}\right) + m_a \cdot f \cdot a \cdot h \cdot \omega_z^2\sqrt{\frac{b^2}{L^2} + ctg^2\bar{\alpha}}\,. \tag{20}$$

From equation (20), we determine:

$$M''_{k_2} - M'_{k_2} = M_{ПОВ} - \frac{m_a \cdot f \cdot a \cdot h \cdot \omega_z^2\sqrt{\frac{b^2}{L^2} + ctg^2\bar{\alpha}}}{\frac{B}{2r_\partial}}\,. \tag{21}$$

The total tangential road reaction acting on the rear wheels R_{k_2} can be determined as follows:

$$R_{k_2} = \frac{M''_{k_2} + M'_{k_2}}{r_\partial} - m_a \cdot g \cdot f\,\frac{a}{L} \tag{22}$$

Equation (4), taking into account relations (5)–(8), (20), and (22), takes the following form:

$$\frac{d\omega_z}{dt}=\frac{2cosec2\bar{\alpha}}{1+\frac{b^2+i_z^2}{L^2}tg^2\bar{\alpha}}\omega_z\frac{d\bar{\alpha}}{dt}+\frac{M_{k_2}^{"}+M_{k_2}^{'}}{m_a\cdot r_\partial\cdot L}\cdot\frac{tg\bar{\alpha}}{1+\frac{b^2+i_z^2}{L^2}tg^2\bar{\alpha}}-\frac{g\cdot f}{L^2}\cdot\frac{a\cdot tg\bar{\alpha}+b\cdot cosec\bar{\alpha}}{1+\frac{b^2+i_z^2}{L^2}tg^2\bar{\alpha}}-$$

$$-\omega_z^2\frac{a\cdot h\cdot f}{L^2}tg\bar{\alpha}\cdot\frac{\sqrt{1+\frac{b^2}{L^2}tg^2\bar{\alpha}}}{1+\frac{b^2+i_z^2}{L^2}tg^2\bar{\alpha}}+\frac{M_{k_2}^{"}-M_{k_2}^{'}}{m_a\cdot r_\partial}\cdot\frac{B}{2L^2}\cdot\frac{tg^2\bar{\alpha}}{1+\frac{b^2+i_z^2}{L^2}tg^2\bar{\alpha}}. \quad (23)$$

Using the method of partial accelerations, we can express equation (23) in the form:

$$\frac{d\omega_z}{dt}=E_{упр.}+E_1-E_2-E_{\omega_z}+E_{МПОВ}, \quad (24)$$

where $\varepsilon_{упр}$ - is the partial angular acceleration caused by the control action $\frac{d\bar{\alpha}}{dt}$ in the steering system,

$$E_{упр.}=\frac{2cosec2\bar{\alpha}}{1+\frac{b^2+i_z^2}{L^2}tg^2\bar{\alpha}}\omega_z\frac{d\bar{\alpha}}{dt}; \quad (25)$$

ε_1 - the partial angular acceleration caused by the action of the driving torques on the rear driving wheels,

$$E_1=\frac{M_{k_2}^{"}+M_{k_2}^{'}}{m_a\cdot r_\partial\cdot L}\cdot\frac{tg\bar{\alpha}}{1+\frac{b^2+i_z^2}{L^2}tg^2\bar{\alpha}}; \quad (26)$$

ε_2 - the partial angular acceleration caused by the rolling-resistance forces acting on the vehicle's wheels,

$$E_2=\frac{g\cdot f}{L^2}\cdot\frac{a\cdot tg\bar{\alpha}+b\cdot cosec\bar{\alpha}}{1+\frac{b^2+i_z^2}{L^2}tg^2\bar{\alpha}}; \quad (27)$$

$\varepsilon_{\omega z}$ - the partial angular acceleration caused by the action of the centrifugal acceleration applied at the vehicle's center of mass,

$$E_{\omega_z}=\omega_z^2\frac{a\cdot h\cdot f}{L^2}tg\bar{\alpha}\cdot\frac{\sqrt{1+\frac{b^2}{L^2}tg^2\bar{\alpha}}}{1+\frac{b^2+i_z^2}{L^2}tg^2\bar{\alpha}}; \quad (28)$$

$\varepsilon_{МПОВ}$ - the additional partial angular acceleration caused by the action of the yaw (turning) moment $M_{ПОВ}$,

$$E_{МПОВ}=\frac{M_{k_2}^{"}-M_{k_2}^{'}}{m_a * r_\partial}\cdot\frac{B}{2L^2}\cdot\frac{tg^2\bar{\alpha}}{1+\frac{b^2+i_z^2}{L^2}tg^2\bar{\alpha}}; \quad (29)$$

To ensure high vehicle handling performance, the dead zone must be eliminated (see Fig. 2). To achieve this, the following condition must be satisfied:

$$\frac{d\omega_z}{dt}=E_{упр.}=\frac{2cosec2\bar{\alpha}}{1+\frac{b^2+i_z^2}{L^2}tg^2\bar{\alpha}}\omega_z\frac{d\bar{\alpha}}{dt}; \quad (30)$$

This is possible when

$$E_1-E_2-E_{\omega_z}+E_{МПОВ}=0 \quad (31)$$

From equation (31), we determine

$$E_{МПОВ}=E_2+E_{\omega_z}-E_1 \quad (32)$$

Substituting expressions (27), (28), and (26) into equation (31), we obtain

$$\varepsilon_{МПОВ}=\frac{g\cdot f}{L^2}\cdot\frac{a\cdot tg\bar{\alpha}+b\cdot cosec\bar{\alpha}}{1+\frac{b^2+i_z^2}{L^2}tg^2\bar{\alpha}}+\omega_z^2\frac{a\cdot h\cdot f}{L^2}tg\bar{\alpha}\frac{\sqrt{1+\frac{b^2}{L^2}tg^2\bar{\alpha}}}{1+\frac{b^2+i_z^2}{L^2}tg^2\bar{\alpha}}-\frac{M_{k_2}^{"}+M_{k_2}^{'}}{m_a\cdot r_\partial\cdot L}\cdot\frac{tg\bar{\alpha}}{1+\frac{b^2+i_z^2}{L^2}tg^2\bar{\alpha}}=$$

$$=\frac{M_{k_2}^{"}-M_{k_2\partial}^{'}}{m_a\cdot r_\partial}\cdot\frac{B}{2L^2}\cdot\frac{tg^2\bar{\alpha}}{1+\frac{b^2+i_z^2}{L^2}tg^2\bar{\alpha}}. \quad (33)$$

From equation (33), we determine the required torque difference between the rear driving wheels

$$\Delta M_k=M_{k_2}^{"}-M_{k_2}^{'}=2\left[\frac{m_a\cdot r_\partial\cdot g\cdot f}{B\cdot tg^2\bar{\alpha}}\left(a\cdot tg\bar{\alpha}+b\cdot cosec\bar{\alpha}\right)+\omega_z^2\frac{m_a\cdot r_\partial\cdot a\cdot h\cdot f}{tg\bar{\alpha}}\sqrt{1+\frac{b^2}{L^2}tg^2\bar{\alpha}}-\frac{L}{B\,tg\bar{\alpha}}\left(M_{k_2}^{"}+M_{k_2}^{'}\right)\right]. \quad (34)$$

Thus, equation (34) has been obtained, which makes it possible to organize the steering control of a rear-wheel-drive vehicle equipped with in-wheel motors under the condition that no "dead-zone–type" nonlinearity is present. To implement condition (34), it is necessary to measure the following parameters:

- the vehicle mass m_a;
- the rolling resistance coefficient f;
- the mean steering angle of the guide wheels $\bar{\alpha}$;
- the coordinates of the vehicle's center of mass a, b, h;
- the driving torques applied to the rear wheels $M'_{K_2}; M''_{K_2}$.

Thus, equation (34) provides a theoretical basis for generating a corrective yaw moment through controlled torque redistribution between the rear wheels, which is consistent with the principles of direct yaw-moment and torque-vectoring control reported in the [7, 8, 10, 13].

4. Determination of the Conditions for Rational Turning Control of a Front-Wheel-Drive Vehicle

Installing in-wheel motors on front-wheel-drive vehicles may lead to instability of the steerable wheels, since a difference in torque between the in-wheel motors can cause their spontaneous rotation, oscillations in the horizontal plane, and a loss of vehicle directional stability. Therefore, the yaw moment should be generated at the rear non-driven wheels by braking the inner wheel (relative to the turning center).

In study [1], physical and mathematical models of the turning process of a wheeled vehicle by a dynamic method—where one of the rear driving wheels is braked—were obtained (see Fig. 1). For the case under consideration, equation (1) remains valid.

However, the tangential reactions R'_{K_2}; R''_{K_2} acting on the rear non-driven wheels during braking of the inner wheel will be directed in the opposite direction (see Fig. 1) and are determined by the following relationships:

$$R'_{k_2} = \frac{M'_{T_2}}{r_\partial} - fR'_{z_2}; \tag{35}$$

$$R''_{k_2} = -fR''_{z_2}; \tag{36}$$

where M'_{T_2} - is the braking torque applied to the inner rear wheel.

Thus, the yaw moment generated when braking the inner rear wheel is equal to:

$$M_{ПОВ} = \frac{B}{2}\left(R''_{k_2} - R'_{k_2}\right) = \frac{B}{2}\left(-fR''_{z_2} + \frac{M'_{T_2}}{r_\partial} + fR'_{z_1}\right) =$$

$$= \frac{B}{2r_\partial}M'_{T_2} - \frac{B}{2}f(R''_{z_2} - R'_{z_1}) \tag{37}$$

Substituting expressions (18) and (19) into equation (36), we obtain:

$$\text{М}_{\text{ПОВ}} = \frac{B}{2r_\partial}M'_{T_2} - 2m_a \cdot a \cdot h \times$$

$$\times\ \omega_z^2\sqrt{1 + \frac{b^2}{L^2} + tg^2\bar{\alpha}\cdot\ ctg\bar{\alpha}} \tag{38}$$

Equation (4), taking into account that in the present case, instead of the expression – $R''_{K_1}\cdot R'_1 - R'_{K_1}\cdot R'_1$ it is necessary to substitute:

$$R_{k_1}R_1 = \frac{L}{sin\bar{\alpha}}R_{f_1} = \frac{m_a\cdot g\cdot f\,\frac{b}{L}}{sin\bar{\alpha}} = m_a\cdot g\cdot f\,\frac{b}{sin\bar{\alpha}}, \tag{39}$$

takes the form:

$$\frac{d}{dt}\left(Y_{zO_2}\cdot\omega_z\right) = R_2\left(R'_{k_2} + R''_{k_2}\right) + M_{ПОВ} + R_1\cdot R_{k_1.} \tag{40}$$

For the rear non-driven wheels:

$$R'_{k_2} + R''_{k_2} = -\frac{M'_{T_2}}{r_\partial} - m_a gf\frac{a}{L} \tag{41}$$

Thus, for the case under consideration, equation (23) takes the following form:

$$\frac{d\omega_z}{dt} = \frac{4cosec2\bar{\alpha}}{1+\frac{b^2+i_z^2}{L^2}tg^2\bar{\alpha}}\omega_z\frac{d\bar{\alpha}}{dt} + \frac{M_{k_1}}{r_\partial\cdot L\cdot m_a}\cdot\frac{tg\bar{\alpha}\cdot sec\bar{\alpha}}{1+\frac{b^2+i_z^2}{L^2}tg^2\bar{\alpha}} + \frac{M'_{T_2}}{r_\partial\cdot L\cdot m_a}\cdot\frac{\frac{B}{2L}tg\bar{\alpha}-1}{1+\frac{b^2+i_z^2}{L^2}tg^2\bar{\alpha}} - \frac{g\cdot f}{L^2}\times$$

$$\times\ tg\bar{\alpha}\,\frac{a+b\cdot cos\bar{\alpha}}{1+\frac{b^2+i_z^2}{L^2}tg^2\bar{\alpha}} - \omega_z^2\,\frac{f\cdot a\cdot h}{L^2}\times tg^2\bar{\alpha}\,\frac{\sqrt{1+\frac{b^2}{L^2}tg^2\bar{\alpha}}}{1+\frac{b^2+i_z^2}{L^2}tg^2\bar{\alpha}}\cdot\omega_z^2 \tag{42}$$

Using the method of partial accelerations, we can express equation (42) in the following form:

$$\frac{d\omega_z}{dt} = \text{E}_{упр.} + \text{E}'_1 - \text{E}_2 - \text{E}_{\omega_z} + \text{E}_{МПОВ}, \tag{43}$$

The partial angular acceleration $\varepsilon_{упр}$ is determined by equation (25). The partial angular acceleration ε_1, caused by the driving torques acting on the front driving wheels of the vehicle, can be determined as follows:

$$E_1 = \frac{M_{k_1}}{m_a \cdot r_\partial \cdot L} \cdot \frac{tg\bar{\alpha} \cdot sec\bar{\alpha}}{1 + \frac{b^2 + i_z^2}{L^2} tg^2\bar{\alpha}} . \quad (44)$$

The partial angular acceleration caused by the rolling-resistance forces acting on the wheels is:

$$E_2 = \frac{g \cdot f}{L^2} tg\bar{\alpha} \cdot \frac{a + b \cdot cos\bar{\alpha}}{1 + \frac{b^2 + i_z^2}{L^2} tg^2\bar{\alpha}} . \quad (45)$$

The partial angular acceleration caused by the action of the centrifugal acceleration applied at the vehicle's center of mass is determined by equation (28).

The additional partial angular acceleration $\varepsilon_{M_{ПОВ}}$, caused by the action of the yaw (turning) moment $M_{ПОВ}$

$$E_{МПОВ} = \frac{M'_{T_2}}{r_\partial \cdot L \cdot m_a} \cdot \frac{\frac{B}{2L} tg\bar{\alpha} - 1}{1 + \frac{b^2 + i_z^2}{L^2} tg^2\bar{\alpha}} . \quad (46)$$

By analyzing dependence (46), it can be concluded that when the acceleration $\varepsilon_{M_{ПОВ}} = 0$ the latter takes the following form:

$$\frac{B}{2L} tg\bar{\alpha} - 1 = 0 . \quad (47)$$

Expression (47) can be written in the following form:

$$\frac{B}{2} = L\,ctg\bar{\alpha} = R_2 . \quad (48)$$

When $\frac{B}{2} < R_2$ the acceleration $\varepsilon_{M_{ПОВ}}$ changes its sign to the opposite, which leads to the neutralization of the yaw moment.

From equation (43), it follows that

$$\frac{d\omega_z}{dt} = E_{упр.} , \quad (49)$$

when the following condition is satisfied

$$E'_1 - E_2 - E_{\omega_2} + E_{МПОВ} = 0 , \quad (50)$$

from which we determine

$$E_{МПОВ} = E_2 + E_{\omega_2} - E'_1 , \quad (51)$$

Taking into account relations (28), (44), and (45), we determine

$$\varepsilon_{МПОВ} = \frac{g \cdot f}{L^2} tg\bar{\alpha} \cdot \frac{a + b \cdot cos\bar{\alpha}}{1 + \frac{b^2 + i_z^2}{L^2} tg^2\bar{\alpha}} + \omega_z^2 \cdot \frac{a \cdot h \cdot f}{L^2} tg\bar{\alpha} \times$$

$$\times \frac{\sqrt{1 + \frac{b^2}{L^2} tg^2\bar{\alpha}}}{1 + \frac{b^2 + i_z^2}{L^2} tg^2\bar{\alpha}} - \frac{M_{k_1}}{r_\partial \cdot L \cdot m_a} \cdot \frac{tg\bar{\alpha} \cdot sec\bar{\alpha}}{1 + \frac{b^2 + i_z^2}{L^2} tg^2\bar{\alpha}} \quad (52)$$

Substituting expression (46) into the left-hand side of equation (52), we obtain:

$$\frac{M'_{T_2}}{r_\partial \cdot L \cdot m_a} \cdot \frac{\frac{B}{2L} tg^2\bar{\alpha} - 1}{1 + \frac{b^2 + i_z^2}{L^2} tg^2\bar{\alpha}} = \frac{g \cdot f}{L^2} tg^2\bar{\alpha} \frac{a + b \cdot cos\bar{\alpha}}{1 + \frac{b^2 + i_z^2}{L^2} tg^2\bar{\alpha}} + \omega_z^2 \frac{a \cdot h \cdot f}{L^2} tg^2\bar{\alpha} \frac{\sqrt{1 + \frac{b^2}{L^2} tg^2\bar{\alpha}}}{1 + \frac{b^2 + i_z^2}{L^2} tg^2\bar{\alpha}} - \frac{M_{k_1}}{r_\partial \cdot L \cdot m_a} \frac{tg\bar{\alpha} \cdot sec\bar{\alpha}}{1 + \frac{b^2 + i_z^2}{L^2} tg^2\bar{\alpha}} . \quad (53)$$

From equation (53), we obtain the required value of the braking torque on the inner rear non-driven wheel

$$M'_{T_2} = \frac{m_a \cdot g \cdot f \cdot r_\partial}{\frac{B}{2} - L \cdot ctg\bar{\alpha}} \left[a + b \cdot cos\bar{\alpha} + \frac{\omega_z^2}{g} \times \right.$$

$$\left. \times h \sqrt{1 + \frac{b^2}{L^2} tg^2\bar{\alpha}} - \frac{M_{k_1} \cdot L}{m_a \cdot g \cdot f \cdot r_\partial} \cdot sec\bar{\alpha} \right] \quad (54)$$

When analyzing expression (54), it can be concluded that, at a turning radius $R = Lctg\alpha$, equal to half the track width $\frac{B}{2}$, the required braking torque M'_{T2} tends to infinity.

5. Determination of the Conditions for Rational Turning Control of an All-Wheel-Drive Vehicle

As previously noted, it is not advisable to install in-wheel motors on the vehicle's front axle, since this may lead to instability of the front wheels and of the vehicle as a whole. The initial dynamic model of the vehicle corresponds to that shown in Fig. 1 and is described by equation (1). In this case, equation (4), taking into account assumption (5), takes the following form:

$$\frac{d}{dt} = \left(I_{Z0_2} \cdot \omega_Z \right) = R_2 \left(R'_{k_2} + R''_{k_2} \right) +$$

$$+ \frac{B}{2} \left(R''_{k_2} - R'_{k_2} \right) + R_{k_2} \cdot R_1 . \quad (55)$$

For a front driving axle

$$R_{k_1} = \frac{M_{k_1}}{\delta_\partial} - f \cdot R_{Z_1} = \frac{M_{k_1}}{\delta_\partial} - m_a \cdot g \cdot f \frac{b}{L} \; . \tag{56}$$

Equation (55) differs from equation (4) by the sign and by expression (56), which is used to determine the total tangential reaction R_{K1} acting on the wheels of the front axle.

Taking expression (56) into account, equation (23) takes the following form:

$$\frac{d\omega_Z}{dt} = \frac{4\mathrm{cosec}2\overline{\alpha}}{1+\frac{b^2+i_Z^2}{L^2}tg^2\overline{\alpha}} \cdot \omega_Z \frac{d\overline{\alpha}}{dt} + \frac{M_{k_2}^{'}+M_{k_2}^{"}}{m_a \cdot \delta_\partial \cdot L} \cdot \frac{tg\overline{\alpha}}{1+\frac{b^2+i_Z^2}{L^2}tg^2\overline{\alpha}} + \frac{M_{k_1}}{m_a \cdot \delta_\partial \cdot L} \cdot \frac{sec\overline{\alpha}}{1+\frac{b^2+i_Z^2}{L^2}tg^2\overline{\alpha}} - \frac{g \cdot f \cdot tg^2\overline{\alpha}}{L^2} \times$$

$$\times \frac{\frac{a}{tg\overline{\alpha}} + \frac{b}{sin\overline{\alpha}}}{1+\frac{b^2+i_Z^2}{L^2}tg^2\overline{\alpha}} - \omega_z^2 \frac{a \cdot h \cdot f}{L^2} tg\overline{\alpha} \cdot \frac{\sqrt{1+\frac{b^2}{L^2}tg^2\overline{\alpha}}}{1+\frac{b^2+i_Z^2}{L^2}tg^2\overline{\alpha}} + \frac{M_{k_2}^{"}+M_{k_2}^{'}}{m_a \cdot \delta_\partial} \cdot \frac{B}{2L^2} \cdot \frac{tg^2\overline{\alpha}}{1+\frac{b^2+i_Z^2}{L^2}tg^2\overline{\alpha}} \, . \tag{57}$$

In this case, the equation of partial accelerations takes the following form:

$$\frac{d\omega_Z}{dt} = \varepsilon_{кер.} + \varepsilon_1^{'} - \varepsilon_2 - \varepsilon_{\omega_Z} + \varepsilon_{мпов} \, , \tag{58}$$

where $\varepsilon_1^{'}$ - is the partial angular acceleration caused by the driving torques acting on the front and rear driving wheels,

$$\varepsilon_1^{'} = \frac{\left(M_{k_2}^{'}+M_{k_2}^{"}\right)tg\overline{\alpha} + M_{k_1}sec\overline{\alpha}}{m_a \cdot \delta_\partial \cdot L\left(1+\frac{b^2+i_Z^2}{L^2}tg^2\overline{\alpha}\right)} \, . \tag{59}$$

From the condition ensuring high vehicle handling given by (30), we determine

$$\varepsilon_{мпов} = \varepsilon_2 + \varepsilon_{\omega_Z} - \varepsilon_1^{'} \; . \tag{60}$$

Substituting expressions (27), (28), and (59) into equation (60), we obtain

$$\varepsilon_{мпов} = \frac{M_{k_2}^{"}-M_{k_2}^{'}}{m_a \cdot \delta_\partial} \cdot \frac{B}{2L^2} \frac{tg^2\overline{\alpha}}{1+\frac{b^2+i_Z^2}{L^2}tg^2\overline{\alpha}} =$$

$$= \frac{gf}{L^2} tg^2\overline{\alpha} \frac{\frac{a}{tg\overline{\alpha}} + \frac{b}{sin\overline{\alpha}}}{1+\frac{b^2+i_Z^2}{L^2}tg^2\overline{\alpha}} + \omega_z^2 \frac{a \cdot h \cdot f}{L^2} \cdot tg\overline{\alpha} \frac{\sqrt{1+\frac{b^2}{L^2}tg^2\overline{\alpha}}}{1+\frac{b^2+i_Z^2}{L^2}tg^2\overline{\alpha}} - \frac{\left(M_{k_2}^{'}+M_{k_2}^{"}\right)tg\overline{\alpha} + M_{k_1}sec\overline{\alpha}}{m_a \cdot \delta_\partial \cdot L\left(1+\frac{b^2+i_Z^2}{L^2}tg^2\overline{\alpha}\right)} \tag{61}$$

From equation (61), we obtain the required torque difference between the rear driving wheels

$$\Delta M_{k_2}^{'} = M_{k_2}^{"} - M_{k_2}^{'} = 2\left[\frac{m_a \cdot \delta_\partial \cdot g \cdot f}{B}\left(\frac{a}{tg\overline{\alpha}} + \frac{b}{sin\overline{\alpha}}\right) + \omega_Z^2 \frac{m_a \cdot \delta_\partial \cdot a \cdot h \cdot f}{tg\overline{\alpha}} \sqrt{1+\frac{b^2}{L^2}tg^2\overline{\alpha}} - \frac{L}{B}\frac{\left(M_{k_2}^{'}+M_{k_2}^{"}\right)tg\overline{\alpha} + M_{k_1}sec\overline{\alpha}}{tg^2\overline{\alpha}}\right]. \tag{62}$$

By comparing expressions (34) and (62), it can be concluded that the magnitude of the torque variation $\Delta M_{K2}^{'} < \Delta M_{K2}$ by the amount

$$x = \frac{2L}{B} \cdot \frac{M_{k_1}sec\overline{\alpha}}{tg^2\overline{\alpha}} \tag{63}$$

The ideal-handling condition (30) can be satisfied when $\Delta M_{K2}^{'} = 0$.

This is possible in the case (see equation (62)).

$$\frac{m_a \cdot \delta_\partial \cdot g \cdot f}{B}\left(\frac{a}{tg\overline{\alpha}} + \frac{b}{sin\overline{\alpha}}\right) + \omega_Z^2 \frac{m_a \cdot \delta_\partial \cdot a \cdot h \cdot f}{tg\overline{\alpha}} \cdot \sqrt{1+\frac{b^2}{L^2}tg^2\overline{\alpha}} - \frac{L}{B}\frac{\left(M_{k_2}^{'}+M_{k_2}^{"}\right)tg\overline{\alpha} + M_{k_1}sec\overline{\alpha}}{tg^2\overline{\alpha}} = 0 \tag{64}$$

from which we determine

$$\left(M_{k_2}^{'}+M_{k_2}^{"}\right)tg\overline{\alpha} + M_{k_1}sec\overline{\alpha} = \frac{m_a \cdot \delta_\partial \cdot g \cdot f}{L} tg^2\overline{\alpha}\left(\frac{a}{tg\overline{\alpha}} + \frac{b}{sin\overline{\alpha}}\right) + \omega_Z^2 \cdot m_a \cdot \delta_\partial \frac{a \cdot h \cdot B \cdot f}{L} tg\overline{\alpha}\sqrt{1+\frac{b^2}{L^2}tg^2\overline{\alpha}} \, . \tag{65}$$

The total driving torque on the rear wheels in this case is

$$M_{k_2} = M^{'}_{k_2} + M^{''}_{k_2} = \frac{m_a \cdot \delta_\partial \cdot g \cdot f}{L} tg\bar{\alpha}\left(\frac{a}{tg\bar{\alpha}} + \frac{b}{sin\bar{\alpha}}\right) +$$
$$+\omega_Z^2 \cdot m_a \cdot \delta_\partial \frac{a \cdot h \cdot B \cdot f}{L}\sqrt{1 + \frac{b^2}{L^2} tg^2\bar{\alpha}} - \frac{M_{k_1}}{sin\bar{\alpha}} \quad (66)$$

For a rear-wheel-drive vehicle $M_{K1} = 0$ and equation (66) takes the form

$$M_{k_2} = M^{'}_{k_2} + M^{''}_{k_2} = \frac{m_a \cdot \delta_\partial \cdot g \cdot f}{L} tg\bar{\alpha}\left(\frac{a}{tg\bar{\alpha}} + \frac{b}{Sin\bar{\alpha}}\right) +$$
$$+\omega_Z^2 \cdot m_a \cdot \delta_\partial \frac{a \cdot h \cdot B \cdot f}{L}\sqrt{1 + \frac{b^2}{L^2} tg^2\bar{\alpha}} \quad (67)$$

Satisfying equalities (66) and (67) for all-wheel-drive and rear-wheel-drive vehicles makes it possible to obtain the ideal handling condition (30), which ensures proportionality between the controlled variable $\frac{d\omega_z}{dt}$ and the input (control) variable $\frac{d\bar{\alpha}}{dt}$. The obtained results show that, during vehicle turning, in addition to $\frac{d\bar{\alpha}}{dt}$, other control signals may also be present—in this case, the driving torques applied to the driven wheels. This means that even when using a purely kinematic steering method, the chassis of the vehicle may have two or more degrees of freedom [3].

This conclusion agrees with recent studies on integrated chassis control, in which torque vectoring is coordinated with supplementary steering functions, including rear-wheel steering, to improve stability, controllability, and fault tolerance [14, 16].

By comparing expressions (66) and (67), it can be concluded that implementing condition (30) with a zero torque difference between the rear wheels on an all-wheel-drive vehicle—compared to a rear-wheel-drive vehicle—can be achieved with a smaller total driving torque at the rear axle. This is particularly important for ensuring the vehicle's trajectory stability and handling performance.

6. Conclusions

As a result of the study, a method has been proposed to improve the handling of a rear-wheel-drive vehicle equipped with in-wheel motors during cornering by eliminating the "dead-zone–type" nonlinearity. To achieve this, a combined control system with two degrees of freedom is proposed, enabling the simultaneous use of both kinematic and dynamic steering methods.

The study has produced an equation that makes it possible to control the turning of a rear-wheel-drive vehicle with in-wheel motors under the condition that no dead-zone–type nonlinearity is present. To implement this steering strategy, it is necessary to measure the vehicle mass, the rolling resistance coefficient *f*, the mean steering angle of the guide wheels, the coordinates of the vehicle's center of mass, and the driving torques applied to the rear wheels.

The study has shown that, for the implementation of a combined steering method on a front-wheel-drive vehicle, it is necessary to brake the inner rear non-driven wheel (relative to the turning center). Creating a torque imbalance on the front steerable and driving wheels is associated with a risk of destabilizing the motion of the front wheels and the vehicle as a whole.

The obtained analytical dependence of the required braking torque on the inner rear wheel on the steering angle of the guide wheels, angular velocity, and the mass and geometric parameters of the vehicle enables further stabilization of combined steering systems for front-wheel-drive vehicles.

Satisfying equalities (66) and (67) for all-wheel-drive and rear-wheel-drive vehicles makes it possible to achieve the ideal-handling condition (30), which ensures proportionality between the controlled variable $\frac{d\omega_z}{dt}$ and the input (control) variable $\frac{d\bar{\alpha}}{dt}$. The results demonstrate that, during vehicle turning, in addition to $\frac{d\bar{\alpha}}{dt}$, other control signals may arise in this case, the driving torques applied to the driven wheels. This implies that even when a purely kinematic steering method is used, the vehicle's chassis may possess two or more degrees of freedom [3].

When the torque difference between the rear wheels is zero for both all-wheel-drive and rear-wheel-drive vehicles, the condition $\frac{d\omega_z}{dt} = \varepsilon_{упр.}$ is satisfied. This condition is achievable with a smaller total driving torque on the rear wheels. This is particularly important for ensuring the vehicle's trajectory stability and handling performance.

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