

# TRANSVERSE DYNAMICS OF MOTION OF A WHEEL TRACTOR WITHOUT ELASTIC SUSPENSION

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**Abstract** - This article examines the lateral dynamics of a wheeled tractor without a spring system operating on uneven ground. The relevance of this study stems from the widespread use of unsprung tractors equipped with large, low-pressure tires in agricultural and transport operations under difficult road conditions, where lateral vibrations significantly impact stability, controllability, and operator working conditions.

Based on an analysis of the design concepts of articulated tractors with a balance-beam front axle, computational models equivalent to their oscillatory systems in the transverse plane were developed. A system of differential equations was developed that describes the interconnected lateral-angular and linear vibrations of the structural elements, taking into account the inertial characteristics and reactions of the ground surface.

The numerical solution of the resulting system of equations was achieved by integrating disturbances simulating the tractor's movement over a single uneven surface. A comparative study of the dynamic behavior of tractors with different design concepts was conducted. It was established that the nature of the coupling between section oscillations significantly depends on the chassis type, as well as the values of the inertial parameters and coupling coefficients.

It was shown that articulated tractors are characterized by weak coupling between section oscillations, while machines with a balance-beam front axle exhibit more pronounced coupling of oscillation processes, resulting in reduced lateral acceleration levels and improved ride comfort.

Experimental studies confirmed the validity of the developed mathematical model. It was found that lateral accelerations are significant and have an unfavorable frequency range, comparable to vertical oscillations, requiring their consideration when assessing operating conditions.

These results allow us to refine the physical picture of lateral oscillations in unsprung wheeled tractors and can be used to validate design parameters and improve the operational reliability of the machines.

**Keywords:** Lateral dynamics; Tractor; Suspension-free system; Lateral oscillations; Motion stability; Mathematical model; Tire-soil interaction.

## 1. Introduction

Wheeled tractors without elastic suspension, equipped with large, low-pressure tires, are widely used in agricultural, transport, and technological operations in off-road conditions, as well as on soft and uneven ground. The design features of these machines, in particular the absence of an extensive spring system and the use of tires with increased deformability, provide increased maneuverability and reduced ground

pressure, which is especially important when operating in difficult road conditions.

However, when driving over uneven terrain, such tractors, in addition to longitudinal-vertical vibrations, experience intense lateral and transverse-angular vibrations. These vibrations have a significant impact on the stability of the vehicle, its controllability, and the operator's working conditions, including vibration load levels and operator comfort. In some cases, lateral vibrations are the determining factor limiting the tractor's

permissible operating modes and speed. It should be noted that the spatial vibrations of unsprung wheeled tractors, particularly in the transverse plane, have not yet been fully studied. Most existing studies focus either on longitudinal-vertical vibrations or on the dynamics of machines with extensive suspension, which does not fully account for the specific performance of unsprung structures, in which the tires serve as the primary elastic-damping element.

The paper [1] examines the lateral stability of an agricultural tractor moving on a slope. A nonlinear dynamic model was developed that takes into account tire-soil interaction, allowing for the determination of the conditions for the transition from stable motion to lateral slip and rollover. The results showed that lateral slip is the dominant factor in stability loss.

The objective of the paper [2] is to study the interaction between longitudinal and lateral slip of a mining tractor. Experimental and simulation studies revealed that the slope angle and traction load significantly influence lateral dynamics, with the effect of speed being nonlinear.

The paper [3] proposes a method for assessing the lateral stability of a tractor with an adjustable body position. A mathematical model has been developed that allows for determining the stability margin depending on the position of the center of mass and the geometric parameters of the machine.

The paper [4] is devoted to the development of a control algorithm for improving the traction and lateral stability of a arable tractor. The proposed adaptive control method reduces lateral slip and improves stability under changing soil conditions. Paper [5] examines the dynamic stability of an agricultural tractor during fieldwork. The authors developed a simulation model to predict the tractor's behavior under variable loads and uneven terrain.

This paper [6] analyzes bifurcation phenomena in the dynamics of a tractor-semi-trailer system. A nonlinear model with multiple degrees of freedom was developed, and critical stability regions were determined depending on the speed and steering angle. Paper [7] studies the influence of external lateral disturbances on vehicle stability. It is shown that, under certain conditions, small disturbances can lead to a loss of stability due to nonlinear effects. Study [8] presents a tire dynamics model that serves as the basis for analyzing the lateral motion of wheeled vehicles. The relationship between lateral forces and slip angles is described using nonlinear relationships. Paper [9] presents a comprehensive analysis of vehicle lateral dynamics, including the influence of tire characteristics, suspension parameters, and road conditions. The paper [10] examines the influence of steering system parameters on vehicle lateral stability and controllability. Frequency characteristics and transfer functions are analyzed.

A sliding mode control algorithm for improving the stability of wheeled vehicles is proposed in [11]. The results demonstrate increased robustness of the system under uncertain operating conditions.

In [12], an adaptive vehicle lateral dynamics control system based on modern optimization methods was developed. Improved tracking accuracy and stability were demonstrated. In [13], a model predictive control (MPC) approach was applied to vehicle lateral dynamics. This method allows for predicting system behavior and preventing instability. In [14], a robust control method for nonlinear dynamic systems, including vehicles operating under uncertainty, was presented. In [15], vibration processes in mechanical systems directly related to the dynamics of unsprung masses were analyzed. In [16], a theoretical model of ground-wheel interaction was developed, which is fundamental for analyzing the dynamics of off-road vehicles. In [17], the mechanics of off-road vehicles, including traction, stability, and interaction with deformable soil, were investigated. In [18], a refined model of tire-ground interaction was proposed, taking into account nonlinear deformation properties. A study [19] analyzes the impact of elastic deformation of a flexible subframe on vehicle ride comfort using mathematical modeling and numerical experiments. It is shown that optimizing the stiffness of subframe bushings using multi-criteria algorithms, particularly genetic algorithms, significantly improves vibration characteristics and enhances ride comfort.

Paper [20] investigates dynamic processes in off-road vehicles using signal processing techniques. In [21], a fuzzy sliding mode control (FSMC) algorithm for vehicle suspension is proposed, which simultaneously ensures road contact stability and ride comfort. Simulations have shown that this approach reduces vibration and prevents wheel lift, outperforming traditional systems, including PID control. Paper [22] examines nonlinear dynamic effects in vehicle motion and identifies critical stability modes. Paper [23] considers multi-link dynamic modeling of mechanical systems used to analyze tractor dynamics. In [24], using a dynamic model and experiments, it was shown that the characteristics of the suspension and tires, including changes in the slip angle under the action of forces, affect the behavior of the car on uneven roads differently depending on the driving speed.

In the work [25], a dynamic model of a railway vehicle was developed taking into account the interaction of wheels and rails, and a robust controller was proposed that ensures increased ride comfort under uncertain parameters and the impact of track unevenness. Study [26] is one of the early studies devoted to the lateral stability of tractors and their attachments, where the oscillatory nature of motion was first identified.

Paper [27] examines the dynamics of vehicle rollovers and identifies critical thresholds. The study [28] analyzes the nonlinear dynamic behavior of mechanical systems and stability loss mechanisms.

The study [29] examines the yaw stability of vehicles, including control methods for improving stability. This paper [30] proposes a method for guaranteed fuzzy steering control based on the Takagi–Sugeno model, which allows the vehicle to remain centered in its lane without knowing the road curvature. Simulations have shown that the combination of  $H^\infty$  and optimal control ensures high driving safety and driving comfort.

An analysis of cited scientific sources [1–30] reveals that the vast majority of modern research in the field of wheeled vehicle dynamics focuses on issues of lateral stability, controllability, propeller-ground interaction, and the development of active control systems. Significant attention is devoted to the construction of nonlinear mathematical models, the analysis of bifurcation phenomena, and the numerical modeling of dynamic processes under extreme operating conditions.

However, it should be noted that available publications do not sufficiently address the lateral dynamics of unsprung wheeled tractors. Most existing studies focus on vehicles with extensive suspension or examine general dynamic patterns without considering the specifics of a rigid (unsprung) structure, in which the tires effectively act as the sole elastic-damping element. This significantly alters the nature of vibration processes and the redistribution of dynamic loads.

Furthermore, the complex influence of lateral vibrations, terrain unevenness, and changing physical and mechanical properties of the soil on the stability of unsprung tractors has not been adequately studied. There is virtually no research aimed at substantiating the rational design parameters of such machines, taking into account dynamic loading and stability criteria.

The aim of this study is to investigate the lateral dynamics of a wheeled tractor without a spring system by developing and analyzing a mathematical vibration model that takes into account the design features of the machine and the impact of uneven ground surfaces. This model also examines the influence of tractor parameters on its stability, controllability, and operator operating conditions.

## **2. Mathematical Model of Transverse Dynamics**

The development of a mathematical model that describes the lateral dynamics of a wheeled tractor without a spring suspension allows for a detailed look at the physical processes that occur during the course of an uneven terrain. In addition to transport vehicles equipped with classic suspension systems, springless

tractors have tires as their main spring and damping elements, the deformability of which significantly affects the dynamic response of the system.

If the tractor rolls over an uneven surface, the leather wheel detects browning caused by uneven surfaces. This drilling generates folding space rocking of the tractor body, including side displacements and transverse (roll-type) rocking. Through the presence of a special suspension system, the transmission of these drillings to the tractor frame is achieved more directly, which leads to greater dynamic force and stronger coupling between different steering modes.

The key feature of lateral dynamics in such systems is the interaction between the side movements and the wrapping sections of the tractor. This interaction is determined not only by the mass and geometric parameters of the machine, but also by the distribution of inertial powers of the main axles. In short, the presence of sub-central (pre-button) moments of inertia leads to a connection between the forward and backward movements, which significantly complicates the dynamic behavior. In articulated tractors or tractors with a front axle type of balancer, the structure can be thought of as a rich body system that consists of interconnected rigid sections. The skin section is built up to the independent level, and its dynamics are connected through kinematic exchanges and force interactions. Therefore, the formulation of a mathematical model requires the introduction of coordinates that describe both the lateral movements and the cutaneous changes in the skin section.

The choice of external coordinates is based on the physical interpretation of the system behavior. On the transverse plane, manually describe the rotation, rotation and side movements of characteristic points (for example, above the front and rear axles) and the rotations of the vertical sections of the front and rear axles of the tractor. This presentation allows for both the formation of a solid body and the flow between sections.

An important aspect of modeling is the description of the interaction between the tires and the supporting surface. In general, this interaction is likely to be characterized by rigidity and damping forces, which lie in the design of the tire, the pressure in the tires and the characteristics of the ground. For small children, the bowel strength can be approximated as proportional to the bowel displacement and kuta forging, which allows for linearization of the position in the first proximity.

When additional folding is introduced onto the surface of the soil, the fragments can demonstrate non-linear and time-dependent behavior. However, for the purposes of analytical modeling, it is completely assumed that the reaction of the soil can be represented as an equivalent spring-damping element that acts at the points of contact. This allowance allows the task to be reduced to a system with moderate parameters, while preserving the particularity of the dynamic response.

The equations of motion are formulated using classical methods of analytical mechanics, such as the Lagrange equations of the second kind or the Newton-Euler equations. These approaches allow for systematic consideration of inertial forces, external loads, and constraint responses. The resulting system of differential equations describes coupled oscillatory processes in the transverse plane.

It should be noted that the lateral dynamics of unsprung tractors are characterized by the presence of multiple natural frequencies corresponding to different motion modes. Typically, low-frequency modes are associated with the lateral movement of the entire machine, while high-frequency modes correspond to the angular oscillations of individual sections. The coupling between these modes leads to energy exchange and can cause complex oscillation patterns under external excitation.

Furthermore, the mass distribution between the front and rear sections plays a crucial role in determining the dynamic characteristics of the system. The shift in the center of mass affects both the inertial properties and the distribution of wheel loads, which in turn affects the lateral stiffness and damping forces. Therefore, the model must explicitly include parameters describing the mass distribution and geometric configuration. Thus, constructing an adequate mathematical model requires considering the following key factors: the rigid structure of the tractor, the inertial characteristics of its sections, the elastic damping properties of the tires, and the interaction with the supporting surface. Based on these considerations, it becomes possible to develop equivalent calculation models that reflect the key features of the transverse vibration system.

Based on the structural analysis, calculation schemes equivalent to the oscillatory systems of unsprung wheeled tractors were developed, as shown in Fig. 1. The calculation scheme in Fig. 1, a, corresponds to a tractor with a hinge between sections, while Fig. 1, b, to a tractor with a balance beam front axle.

For the case of rectilinear tractor motion at a constant speed, the differential equations for oscillation in the transverse plane will be [31].

$$\left. \begin{aligned} M_1 \ddot{y}_1 + M_3 \ddot{y}_2 + \frac{m_1 h_1 (l_2 + c_1) + J_{01}^{xz}}{L} \ddot{\beta}_1 + \frac{m_2 h_2 (l_2 - c_2) + J_{02}^{xz}}{L} \ddot{\beta}_2 &= P_{1l}^y + P_{1r}^y; \\ J_{1l}^{xz} \ddot{\beta}_1 + \frac{m_1 h_1 (l_2 + c_1) + J_{01}^{xz}}{L} \ddot{y}_1 + \frac{m_1 h_1 (l_1 - c_1) + J_{01}^{xz}}{L} \ddot{y}_2 &= (P_{1l}^z - P_{1r}^z)^\delta - \\ &- (P_{1l}^y + P_{1r}^y)^\delta + G_1 h_1 \beta_1; \\ M_2 \ddot{y}_2 + M_3 \ddot{y}_1 + \frac{m_2 h_2 (l_1 + c_2) - J_{02}^{xz}}{L} \ddot{\beta}_2 + \frac{m_1 h_1 (l_1 - c_1) - J_{01}^{xz}}{L} \ddot{\beta}_1 &= P_{2l}^y + P_{2r}^y; \\ J_{2l}^{xz} \ddot{\beta}_2 + \frac{m_2 h_2 (l_1 + c_2) - J_{02}^{xz}}{L} \ddot{y}_2 + \frac{m_2 h_2 (l_2 - c_2) + J_{02}^{xz}}{L} \ddot{y}_1 &= (P_{2l}^z - P_{2r}^z)^\delta - \\ &- (P_{2l}^y + P_{2r}^y)^\delta + G_2 h_2 \beta_2, \end{aligned} \right\} (1)$$

where  $y_1$  and  $y_2$  are the horizontal transverse displacements of points 1 and 2 above the front and rear axles;  $\beta_1$  and  $\beta_2$  are the transverse-angular oscillations

of the front and rear sections of the tractor;  $M$  is the mass of the tractor;  $m_1$ ,  $m_2$  and  $G_1$ ,  $G_2$  are the masses and weights of the front and rear sections;  $J_1^{xx}$ ,  $J_2^{xx}$  are the moments of inertia of the front and rear sections relative to the axis of the horizontal hinge;  $J_{01}^{xz}$ ,  $J_{02}^{xz}$  are the centrifugal moments of inertia of the sections relative to the axes parallel to the axes  $XZ$  and passing through the centers of mass of the sections;  $P_i^z$ ,  $P_i^y$  are the vertical and lateral reactions of the track surface acting on the tractor wheels and depending on the parameters of the system and the profile of irregularities;  $M_1$ ,  $M_2$  and  $M_3$  are the reduced masses.

The system of equations (1) reflects the interconnected nature of the lateral and angular oscillations of the tractor sections. The terms containing inertial parameters (masses and moments of inertia) describe the system's resistance to acceleration, while the coupling terms, including the centrifugal moments of inertia  $J_{xz}$ , characterize the interaction between the translational and rotational motions.

Physically, the presence of non-zero centrifugal moments of inertia leads to an additional dynamic coupling between the lateral displacements and angular oscillations of the tractor body. This effect becomes especially significant for machines with distributed mass and asymmetric loading conditions.

The coupling coefficients between the front and rear sections depend on the geometric parameters of the tractor and the relative positions of their centers of mass. Reducing these coefficients leads to a weakening of the interaction between the sections and, consequently, to more independent oscillatory behavior of each machine part.

Several simplifying assumptions were made in developing the mathematical model. In particular, small angular displacements were assumed, which allowed the governing equations to be linearized. The interaction between the tires and the ground was considered in a quasi-static regime, ignoring high-frequency soil deformation effects. Furthermore, the elastic properties of the tires were assumed to be linear over the considered range of operating conditions.

The assumptions made primarily affect the quantitative accuracy of the model, while maintaining its qualitative ability to describe the main dynamic effects. In particular, the linearization of the equations allows the identification of the main vibration modes and the assessment of their interaction without introducing excessive mathematical complexity.

Despite the simplification, the developed model retains sensitivity to key design parameters, such as mass distribution, geometric configuration and tire stiffness. This makes it a useful tool for comparative analysis of different tractor configurations and the identification of trends in dynamic response under different operating conditions.

In further studies, the model can be improved by including nonlinear tire characteristics, variable soil properties and dissipative effects associated with energy losses in the contact patch. However, even in its current form it provides a sufficiently accurate representation of the transverse dynamics for engineering analysis and design evaluation.

$M_1$ ,  $M_2$  and  $M_3$  calculated using the formulas:

$$M_1 = M \frac{l_2^2 + \rho_z^2}{L^2}; M_2 = M \frac{l_1^2 + \rho_z^2}{L^2}; M_3 = M \frac{l_1 l_2 + \rho_z^2}{L^2}; \quad (2)$$

$\rho_z$  – the radius of inertia of the tractor relative to the vertical axis passing through its center of mass. The remaining notations are clear from Fig. 1. The system of differential equations (1) corresponds to the design scheme of a tractor with a universal joint between its sections. For a tractor with a balance beam front axle, the sign  $h_1$  in the equations should be reversed.

The reduced mass  $M_3$  characterizes the relationship between the linear transverse oscillations of the tractor

points above the front and rear axles when the body rotates relative to the axis  $Z$  passing through its center of mass. There is no relationship for  $M_3 = 0$ , that is,  $\rho_z^2 = l_1 l_2$ .

The coupling coefficients  $m_1 h_1 \frac{(l_1 - c_1) - J_{01}^{xz}}{L}$  and  $m_2 h_2 \frac{(l_2 - c_2) J_{02}^{xz}}{L}$ , characterizing the relationship between the oscillations of the front and rear sections, have different values for tractors with different chassis types. For a tractor with a universal joint, the quantities  $l_1 - c_1$  and  $l_2 - c_2$  are small, and when the centrifugal moments of inertia  $J_{01}^{xz}$  and  $J_{02}^{xz}$  are small, as well as when  $M_3 = 0$ , the equations for the lateral oscillations of the front and rear sections become independent:

$$\left. \begin{aligned} M_i \ddot{y}_i + m_i h_i \frac{l_k + c_i}{L} \ddot{\beta}_i &= P_{il}^y + P_{ir}^y; \\ J_i^{xz} \ddot{\beta}_i + m_i h_i \frac{l_k + c_i}{L} \ddot{y}_i &= (P_{il}^z - P_{ir}^z)^b - (P_{il}^y + P_{ir}^y)^d + G_i h_i \beta_i; \end{aligned} \right\} \quad (3)$$

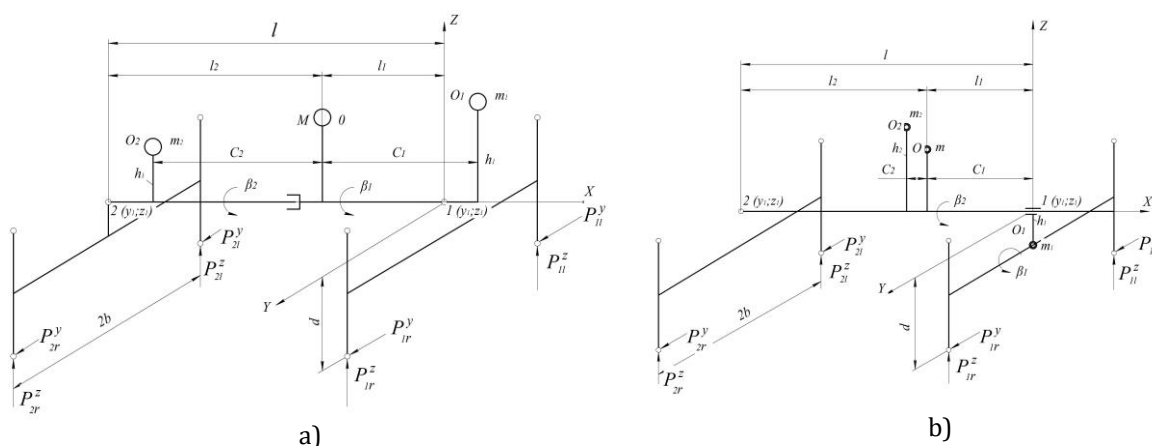


Figure 1: Calculation diagram equivalent to the oscillatory system of a wheeled unsprung tractor: a – tractor with a universal joint between sections; b – tractor with a balance beam front axle

For the front section  $i=1; k=2$ , for the rear  $i=2; k=1$ . For a tractor with a balance front axle  $l_2 - c_2 \neq 0$ , and therefore all the equations of system (1) are interconnected, and the nature of the relationship between individual types of oscillation depends on the specific parameters of the machine. The magnitude of the coupling coefficients in equations (1) also has a significant effect on the position of the instantaneous centers of lateral oscillations of the tractor [32].

### 3. Numerical Modeling Technique

The system of differential equations was solved using numerical integration under a disturbance equivalent to the tractor driving over an isolated cosine-shaped unevenness with one of its wheels. The unevenness dimensions – 1 m long and 10 cm high – were chosen to

account for vibration disturbances in the frequency range relevant to the tractor.

The numerical solution of the system of differential equations was performed using the fourth-order Runge-Kutta method, which provides sufficient accuracy and stability for solving coupled nonlinear dynamic systems. The time step was selected based on the highest natural frequency of the system to ensure numerical convergence and prevent solution instability.

Particular attention was paid to verifying the numerical procedure. The stability of the resulting solution was verified by reducing the integration step and comparing the results. It was found that further reducing the time step leads to minor changes in the calculated accelerations, confirming the adequacy of the chosen numerical parameters.

Vibration calculations were performed for a modern wheeled tractor such as the John Deere 8R,

the design parameters of which were based on experimental and operational data.

The tractor has a balance-beam front axle with suspension components and is equipped with modern wide-profile radial tires, for which test results show that the lateral stiffness is significantly lower than the radial stiffness.

For comparison, the lateral vibrations of a tractor with similar parameters, but with an articulated frame, typical of this type of machine, were also calculated.

The results of the solutions are shown in Fig. 2. For a tractor with a universal joint between sections (Fig. 2, a), when the front axle wheel goes over an uneven surface, intense transverse-angular oscillations of the front section  $\beta_1$  are generated, while oscillations of the rear section  $\beta_2$  are very small, indicating a weak connection between the transverse oscillations of the sections. Horizontal transverse accelerations in the cab at the driver's head level  $\ddot{y}_e$  are determined primarily by transverse-angular accelerations  $\ddot{\beta}_1$ , as can be seen from a comparison of the graphs  $\ddot{\beta}_1$  and  $\ddot{y}_e$ . The vertical accelerations above the front axle  $\ddot{z}_1$  were also calculated in the study.

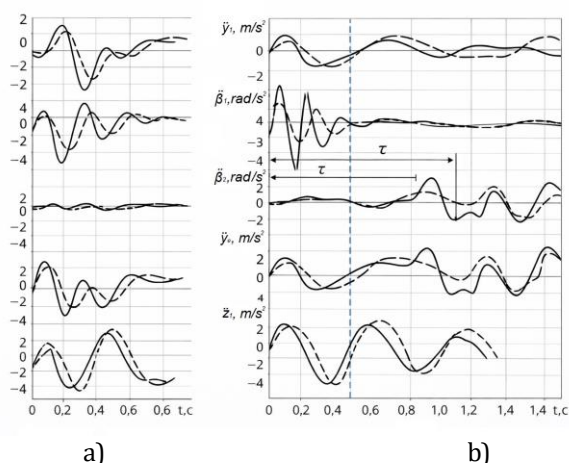


Figure 2. Calculated values of vibrations of an unsprung tractor when driving over an isolated unevenness with the wheels of the left side: - - - -  $V = 3$  m/s; - - - - -  $4$  m/s;  $\tau$  – the time from the start of movement until the rear wheel meets the unevenness (a and b are the same as in Fig. 1)

Figure 2, a shows that accelerations  $\ddot{z}_1$  and  $\ddot{\beta}_1$  are in phase when driving over an uneven surface. Therefore, vertical accelerations under the driver's seat, which is usually offset from the longitudinal plane of symmetry of the vehicle, increase due to lateral oscillations.

The lateral oscillations of a tractor with a balance-beam front axle behave differently (Figure 2, b). When driving over an uneven surface, the front axle wheel generates intense transverse oscillations of the front axle  $\beta_1$ , but transverse oscillations of the body  $\beta_2$  are virtually absent.

Tractor body vibrations  $\beta_2$  are excited when the rear axle wheel passes over an uneven surface, but by this time the vertical accelerations above the front axle  $\ddot{z}_1$ , where the driver's cab is usually located, have had time to decrease and, moreover, are out of phase with the transverse-angular accelerations  $\ddot{\beta}_2$ . Thus, the vertical accelerations under the driver's seat are weakened. Moreover, Fig. 2 shows that both the transverse-angular  $\ddot{\beta}_2$  and horizontal transverse accelerations  $\ddot{y}_1$  of the body of a tractor with a balance-beam front axle have smaller values than the corresponding accelerations  $\ddot{\beta}_1$  and  $\ddot{y}_1$  of the front section of a tractor with a universal joint, which is explained by the higher value of the moment of inertia in the transverse plane of a tractor with a balance-beam axle. Thus, for the considered case of crossing an isolated unevenness, a tractor with a balance front axle has an advantage in terms of ride smoothness [33-35].

The lateral vibrations of a tractor with laterally elastic tires are characterized by two significantly different natural frequencies: horizontal lateral vibrations, which primarily affect machine control, have a low frequency, while transverse-angular vibrations, which primarily affect ride smoothness, have a high frequency. There is a strong coupling between these vibration modes, such that the excitation of one mode is accompanied by the appearance of the other. Furthermore, this coupling affects the values of the natural frequencies. Figure 2 shows that horizontal lateral accelerations  $\ddot{y}_e$  at the driver's head, which are largely dependent on transverse-angular accelerations, have a frequency content (3–5 Hz) that is unfavorable for the human body and are close in magnitude to vertical accelerations.

To assess the impact of key design and operational parameters on the tractor's lateral dynamic response, a parametric analysis of the developed mathematical model was conducted.

The results showed that tire lateral stiffness has a significant impact on the natural frequencies and amplitudes of vibrations. A decrease in stiffness leads to a decrease in the natural frequencies and an increase in the amplitudes of vibrations, which can cause resonance in the frequency range critical for operator comfort.

Weight distribution between the front and rear sections also plays a significant role. Increasing the mass of the front section leads to increased inertial drag and alters the phase relationship between angular and translational vibrations.

The effect of driving speed was also analyzed. It was found that increasing speed leads to a significant increase in dynamic loads due to a higher excitation velocity when overcoming terrain irregularities. This effect is particularly pronounced for tractors with

articulated frames, where the coupling between sections is weaker.

Furthermore, the height and length of the surface irregularities affect the spectral composition of the excitation. Shorter, sharper bumps produce high-frequency vibrations, while longer bumps primarily affect low-frequency modes associated with the overall motion of the vehicle.

The obtained results can be explained by the characteristics of mass and inertia distribution, as well as the tractor's structural configuration. In the case of a front axle with a balance beam, an increase in the transverse moment of inertia leads to a redistribution of dynamic energy, reducing the amplitude of angular vibrations transmitted to the tractor body.

The observed phase shift between vertical and transverse-angular vibrations plays a crucial role in improving ride comfort. When these vibrations are out of phase, partial compensation of accelerations occurs, resulting in a reduction in the overall vibration load experienced by the operator.

In contrast, for tractors with articulated frames, the weak coupling between the sections leads to a concentration of vibration energy in the front section, which directly impacts the operator's workstation. This explains the higher levels of lateral and angular accelerations observed in this configuration.

#### 4. Experimental Research

Experimental studies of the tractor were also conducted. Experimental values of the tractor's natural transverse oscillation frequencies were determined from recordings of the relative transverse-angular oscillations of the tractor body and balance axle  $\Delta\beta = \beta_2 - \beta_1$ . A sample oscillogram is shown in Fig. 3. The experimentally determined frequencies are close to the theoretical values.

Experimental measurements were conducted using accelerometers installed at key points throughout the tractor's structure, including the operator's seat and front axle. Prior to testing, the measurement system was calibrated to ensure accuracy.

Comparison of the experimental and theoretical results showed satisfactory agreement, with discrepancies in key dynamic parameters not exceeding 10-15%. These differences can be explained by simplifications adopted in the mathematical model, as well as variability in soil properties and measurement noise.

The obtained results confirm the validity of the developed model and its suitability for analyzing the lateral dynamics of unsprung wheeled tractors under real-world operating conditions.

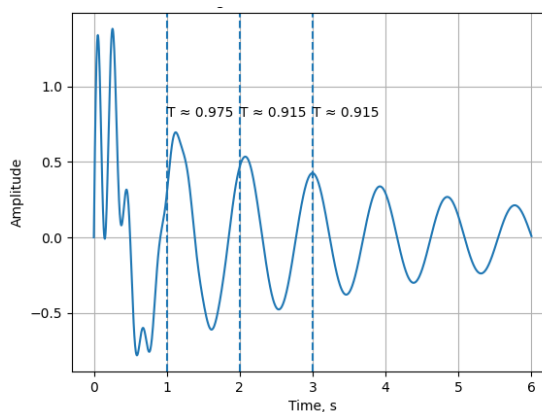


Figure 3. Oscillogram of free transverse-angular oscillations of a tractor

The vertical and horizontal lateral accelerations were measured while the tractor was moving on an uneven dirt road. The results showed that the root-mean-square values of horizontal lateral accelerations  $\ddot{y}_e$  were 15–25% lower than the vertical ones.

Thus, the proposed equations (1) allow us to clarify the physical picture of lateral vibration processes in wheeled unsprung vehicles, as well as analyze the influence of design parameters and operating conditions on them. The data presented suggests that it is advisable to evaluate the ride quality of this type of tractor when driving off-road by taking into account the combined effect of both vertical and lateral accelerations.

#### 5. Conclusions

Theoretical and experimental studies of the lateral dynamics of a wheeled tractor without elastic suspension have established patterns in the formation of lateral and transverse-angular oscillations when moving over uneven ground.

The developed mathematical model, based on a system of differential equations, adequately describes the interconnected oscillatory processes of the tractor's structural elements, taking into account inertial parameters and the interaction of the propellers with the soil, as confirmed by experimental studies.

Numerical modeling revealed that when overcoming a single inequality 1 m long and 0.1 m high in the speed range of 3–4 m/s, intense lateral and transverse-angular oscillations occur in the system, characterized by the presence of two significantly different frequencies. Furthermore, horizontal lateral accelerations have a frequency range of 3–5 Hz, which is unfavorable in terms of vibrational impact on the operator. Experiments have shown that the root-mean-square values of horizontal lateral accelerations are 75–85% of the vertical accelerations, indicating their significant influence on the formation of vibration loads and the need to take them into account when assessing ride quality.

It has been demonstrated that the design of the chassis significantly influences the nature of vibration processes. Specifically, articulated tractors are characterized by weak coupling of the vibrations of the sections, whereas machines with a balance front axle exhibit closer coupling of vibrations, resulting in reduced levels of lateral-angular and horizontal accelerations, as well as partial phase misalignment of the vibrations, thereby reducing the dynamic load.

These results allow us to clarify the physical picture of lateral vibrations in unsprung wheeled tractors and can be used to substantiate rational design parameters for the machines, increase their operational reliability, and improve operator working conditions.

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