

NATURAL FREQUENCY ANALYSIS AND PARAMETERS OPTIMIZATION OF 3-(PRRPR)RC ORIENTATION-ADJUSTING AND VIBRATION-ISOLATION PLATFORM

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Abstract - To suppress the combined disturbances of large-angle pose variations and medium-to-high frequency vibrations on onboard equipment, the 3-(PRRPR)RC active-passive orientation-adjusting and vibration-isolation platform is adopted. However, previous studies have only solved the natural frequencies at the initial pose, and the selection of key structural parameters has long relied on empirical experience. Based on the existing model, this paper establishes a global variation and sensitivity model of natural frequencies, and takes the dynamic performance index and natural frequency as collaborative objectives, employing a genetic algorithm for multi-objective optimization. The results show that the optimal parameters can effectively reconcile active pose-adjustment accuracy and passive vibration isolation bandwidth, and the overall performance outperforms the empirical scheme, providing a quantitative basis for the design of similar platforms.

Keywords: Parallel platform, Vibration isolation, Natural frequency, Parameter optimization.

1. Introduction

Instruments and equipment mounted on carriers are susceptible to large pose variations and medium-to-high frequency vibrations. How to effectively compensate for these two types of disturbances and provide a stable working environment for onboard devices has become a hot research topic [1-2].

For large-amplitude pose variations, Mishra et al. [3] adopted a 2PRU-1PRS configuration to suppress roll, pitch and vertical translational motions; the Dutch company Barge Master designed an offshore cargo-receiving stable platform based on the 3SS-3SPS mechanism [4]. Masoud et al. [5] developed a 3-DOF stabilization device suitable for both ground and aerial carriers. Meanwhile, for multi-dimensional vibration isolation, three strategies including active vibration isolation (mainly for low frequency), passive vibration isolation (mainly for medium-to-high frequency) and hybrid active-passive vibration isolation (broadband) have been

systematically studied [6-8]. Since large-angle pose variations and high-frequency vibrations often coexist in practical working conditions, hybrid-configuration platforms have attracted widespread attention in recent years. For example, Zhang et al. [9] proposed a hybrid active-passive vibration isolation platform consisting mainly of a 4-UPS/CPC parallel mechanism, with hydraulic actuators and a spring-damper system connected in parallel. Geng et al. [10] integrated the actuators and passive vibration isolation units into the closed-loop subchains and proposed a 3-(PRRPR)RC active-passive orientation-adjusting and vibration-isolation platform.

For the passive units in this platform that are tasked with medium-to-high frequency vibration isolation, according to vibration isolation theory, the isolator enters the effective isolation region only when the excitation frequency is greater than $\sqrt{2}$ times the natural frequency of the system.

Therefore, the natural frequency is a core parameter for evaluating passive vibration isolation performance. Regarding the pose dependence of the natural frequency of parallel mechanisms, a number of studies have been conducted. Zhao et al. [11] established vibration differential equations based on the dynamic model of a 3-P(4S) parallel platform and analyzed the variation characteristics of natural frequency with motion positions. Yuan et al. [12] established the parametric vibration equation of a 3UPS/S stabilizing platform, revealing the variation characteristics of both the natural frequency and the vibration response with respect to the mechanism pose. Zhu et al. [13] investigated the relationship between the static stiffness and the natural frequency of a 3-PRR fully compliant parallel mechanism. These studies indicate that the natural frequency of parallel mechanisms is highly sensitive to the pose state, making it necessary to analyze the variation law over the entire workspace. However, for the 3-(PRRPR)RC active-passive orientation-adjusting and vibration-isolation platform proposed in Ref. [10], although its dynamic model has been established and the natural frequencies at the initial pose have been solved, the global variation law of the natural frequency with pose remains unclear, and the sensitivity of key structural parameters to the natural frequency has not yet been quantified. Consequently, it is difficult to provide systematic guidance for subsequent vibration isolation bandwidth design and parameter optimization.

Furthermore, when the orientation-adjusting and vibration-isolation platform performs large-amplitude pose-adjustment tasks, the inertial forces of the payload, the moving platform, and the connecting links are mapped through the mechanism coupling relationships to the elastic elements of the passive vibration isolation units, inducing additional deformations that directly affect the pose accuracy of the moving platform. To reduce these deformations and thereby improve pose-adjustment accuracy, one would intuitively increase the stiffness of the elastic elements; however, increasing the stiffness inevitably raises the natural frequency of the system, thereby narrowing the effective bandwidth of the passive vibration isolation. It follows that there is an inherent design trade-off between active pose-adjustment accuracy and passive vibration isolation bandwidth for this platform, and how to reconcile these two conflicting requirements is crucial for realizing its overall performance.

For such multi-objective coupling problems, parameter optimization has proven to be an effective solution. Several fruitful investigations have been carried out. Wu et al. [14] employed particle swarm optimization to optimize the inverse position solution of a redundant parallel mechanism with the natural frequency as the objective. Zheng et al. [15] utilized a genetic algorithm to optimize the

geometric parameters of a 3-PCR vibration-isolation platform aiming at equivalent stiffness. Lu et al. [16] focused on maximizing the workspace and conducted parameter design for a foldable 3-R(US&SPU) mechanism. Pan et al. [17] proposed a hybrid multi-objective evolutionary algorithm and applied it to the dimensional multi-objective optimization of a 2RPU-RPS mechanism. These studies provide useful references for the parameter optimization of parallel mechanisms. For the 3-(PRRPR)RC orientation-adjusting and vibration-isolation platform proposed in Ref. [10], however, the structural parameters are still selected empirically, and a systematic optimization specifically addressing the inherent conflict between pose-adjustment accuracy and vibration-isolation bandwidth has not yet been performed; as a result, the trade-off between the two remains difficult to achieve, which limits further improvement of the platform's overall performance.

In view of this, based on the work of Ref. [10], this paper first systematically investigates the global variation law of the natural frequency of the 3-(PRRPR)RC platform with respect to pose, and establishes a sensitivity model of the natural frequency to the key structural parameters in order to identify the primary and secondary influences of each parameter. Then, taking the dynamic index reflecting the deformation of the elastic elements and the natural frequency characterizing the isolation performance as collaborative optimization objectives, a multi-objective optimization model is formulated according to the specific characteristics of this configuration. By means of a genetic algorithm, a set of optimal structural parameters is obtained that balances pose-adjustment accuracy and vibration-isolation bandwidth, thereby providing a reference for the engineering design of similar hybrid orientation-adjusting and vibration-isolation platforms.

2. Vibration Model of the 3-(PRRPR)RC Mechanism

2.1 Mechanism Description and Dynamic Model

Fig. 1 shows the prototype of the orientation-adjusting and vibration-isolation platform, and Fig. 2 presents the branch schematic diagram of the mechanism. The mechanism can be decomposed into two operating modes: orientation-adjusting and vibration-isolation. During vibration-isolation, the electric cylinder is assumed to be instantaneously locked and acts as a rigid member. The establishment of the coordinate system and the definition of structural parameters in the mechanism schematic have been elaborated in Ref. [10]. This

mechanism possesses three translational degrees of freedom along x_p , y_p , and z_p respectively. In the passive vibration isolation mode, the dynamic equation of the mechanism in joint space can be expressed as [10]

$$\tau_A = \sum M_k^T \ddot{q} + \dot{q}^T \sum H_k^T \dot{q} - (G_q^p)^T W_p^F - \sum (G_q^k)^T W_k^{G(i)} - \tau_E \quad (1)$$

Where τ_A denotes the generalized driving force, which is virtually applied at prismatic joint P_1 , and its actual driving force is always zero. $\sum M_k^T \ddot{q}$ is the superimposed system mass matrix, $\dot{q}^T \sum H_k^T \dot{q}$ is the Coriolis term (second-order nonlinear coupling term), $(G_q^p)^T W_p^F$ is the generalized driving force generated by external loads, $(G_q^k)^T W_k^{G(i)}$ is the generalized driving force due to gravity of the mechanism links, and τ_E is the elastic force of the spring. q is the generalized coordinate, $\dot{q}$ is the generalized velocity, and $\ddot{q}$ is the generalized acceleration.

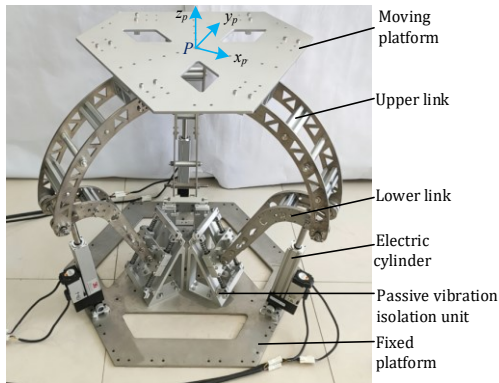


Figure 1: Prototype of the orientation-adjusting and vibration-isolation platform

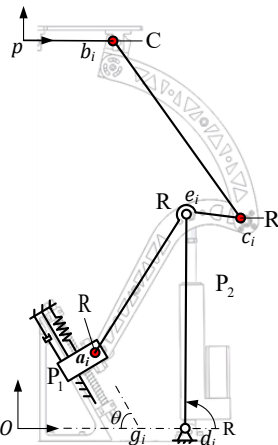


Figure 2: Schematic diagram of the mechanism branch

Under the passive vibration isolation mode, the electric cylinders are assumed to be instantaneously locked and regarded as rigid members. The generalized driving force acting on the driving joint P_1 is set to $\tau_A = 0$. As a constant term, gravity does not affect the calculation results of natural frequencies and can thus be neglected, i.e., $(G_q^k)^T W_k^{G(i)} = 0$. The orientation-adjusting and vibration-isolation platform is in a state of free vibration without external loads, so $(G_q^p)^T W_p^F = 0$. The Coriolis term is related only to velocity $\dot{q}$ and has little influence on natural frequencies; therefore, $\dot{q}^T \sum H_k^T \dot{q}$ can also be ignored in the calculation.

The final dynamic model is simplified as:

$$M\ddot{q} + Kq = 0 \quad (2)$$

Where M is the 3×3 mass matrix $\sum M_k^T$, including the equivalent masses of the moving platform, limbs, sliders and electric cylinders. K is the 3×3 stiffness matrix, which mainly consists of the spring stiffness of the three PRRPR subchains.

2.2 Natural Frequencies and Numerical Examples

For a linear system without damping and external excitation, the natural frequencies of the system are generally solved using the general formula. Assume the free vibration solution is

$$q(t) = u e^{i\omega t} \quad (3)$$

Where, u is the eigenvector. Substituting equation (3) into equation (2) yields

$$(-\omega^2 M + K)u = 0 \quad (4)$$

In order to have a non-trivial solution, the characteristic equation must be satisfied

$$\det(K - \omega^2 M) = 0 \quad (5)$$

Equation (5) is a cubic polynomial equation. When the characteristic roots are distinct, solving it yields three distinct eigenvalues ω_{ni}^2 ($i=1,2,3$), which are the three natural frequencies of the system, and then the corresponding natural modes u_i ($i=1,2,3$) can be obtained.

The structural parameters of the mechanism are listed in Table 1. Substituting these parameters into the above yields the natural frequencies of the mechanism under its initial pose. Under the initial

pose of the mechanism, the first natural frequency is 5.58Hz, the second natural frequency is 5.58Hz, and the third natural frequency is 6.41Hz.

Table 1. Structural parameters of the orientation-adjusting and vibration-isolation platform

Parameters	Value
Initial operating height of the platform, h_0	0.56 m
Circumradius of hinge points, b_i	0.18 m
Circumradius of the fixed platform and electric cylinder, d_i	0.29 m
Circumradius of fixed points, g_i	0.22 m
Length of upper Link, l_{bc}	0.33 m
Length of lower Link, l_{ac}	0.32 m
Angle between prismatic pair and horizontal plane, θ	60°
Spring stiffness value, k	4000N/m

3. Natural Frequency Analysis of the Mechanism

3.1 Analysis of Position-Dependent Natural Frequencies of the Mechanism

As shown in Fig. 3, when the orientation-adjusting and vibration-isolation platform moves in the Z-direction, its natural frequencies of each order increase as the mechanism descends from the initial position, and decrease as the mechanism ascends from the initial position.

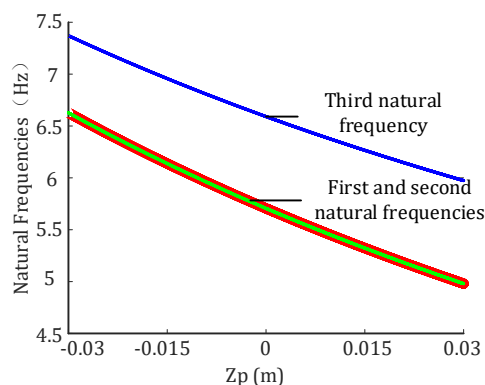
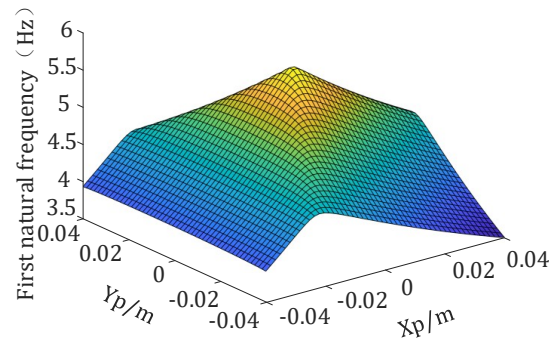
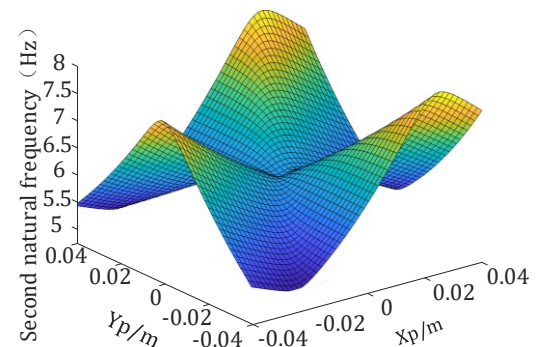


Figure 3: Natural frequency variation in Z-direction motion

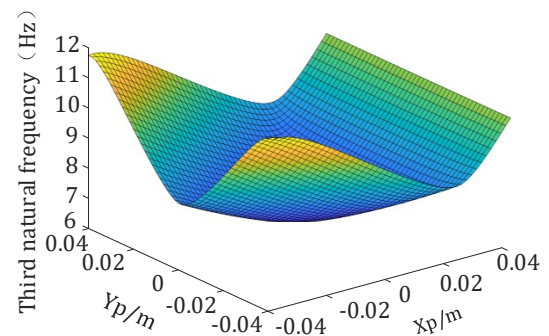
Assuming that the mechanism maintains a constant initial height and moves only in the horizontal plane, the variation of its natural frequencies is shown in Fig. 4.



a) First natural frequency



b) Second natural frequency



c) Third natural frequency

Figure 4: Variation of natural frequencies of each order

As can be seen from Fig. 3 and Fig. 4, the third natural frequency of the orientation-adjusting and vibration-isolation platform is higher than the first and second natural frequencies. According to vibration theory, the lower the natural frequency of the system, the lower its initial vibration isolation frequency, thus enabling a wider effective vibration isolation bandwidth. Therefore, reducing the relatively high third natural frequency can effectively broaden the vibration isolation bandwidth of the platform.

3.2 Sensitivity Analysis of Natural Frequencies

Multiplying both sides of equation (4) by $\mathbf{u}_i^T$ on the left yields

$$\mathbf{u}_i^T (\mathbf{K} - \omega_{ni}^2 \mathbf{M}) \mathbf{u}_i = 0 \quad (6)$$

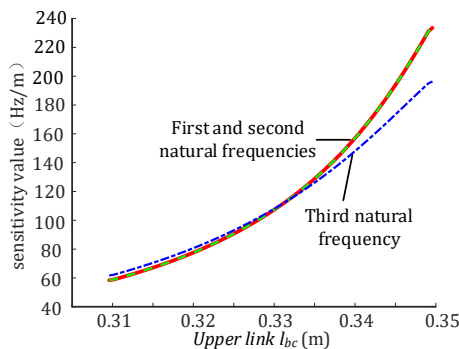
where ω_{ni} and $\mathbf{u}_i^T$ denote the i -th natural frequency of the system and its corresponding mode shape vector, respectively. Let l be a variable encompassing the structural parameters of the mechanism. Differentiating both sides of equation (6) with respect to l yields

$$\begin{aligned} \frac{\partial \mathbf{u}_i^T}{\partial l} (\mathbf{K} - \omega_{ni}^2 \mathbf{M}) \mathbf{u}_i + \mathbf{u}_i^T \left(\frac{\partial \mathbf{K}}{\partial l} - \frac{\partial (\omega_{ni}^2 \mathbf{M})}{\partial l} \right) \mathbf{u}_i \\ + \mathbf{u}_i^T (\mathbf{K} - \omega_{ni}^2 \mathbf{M}) \frac{\partial \mathbf{u}_i}{\partial l} = 0 \end{aligned} \quad (7)$$

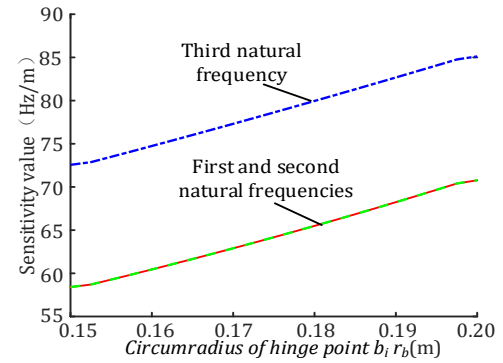
Expanding and simplifying equation (7) allows us to derive the sensitivity expression of the natural frequency of the mechanism

$$\frac{\partial \omega_{ni}}{\partial l} = -\frac{\omega_{ni}}{2} \frac{\mathbf{u}_i^T (\partial \mathbf{M} / \partial l) \mathbf{u}_i}{\mathbf{u}_i^T \mathbf{M} \mathbf{u}_i} \quad (8)$$

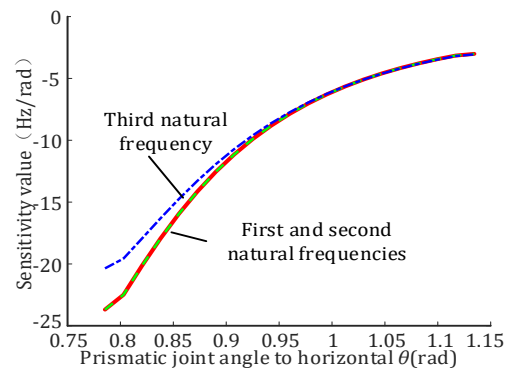
As can be seen from equation (8), the structural parameters of the mechanism have different degrees of influence on its vibration performance. By calculating the first-order partial derivative of each structural parameter with respect to the natural frequency, the natural frequency sensitivity corresponding to each parameter can be quantitatively obtained. On this basis, the key parameters dominating the variation of natural frequency characteristics can be identified, which provides a reliable theoretical basis for subsequent structural optimization and performance improvement. When the mechanism is located at the initial position, the sensitivities of natural frequency with respect to each structural parameter are shown in Fig. 5.



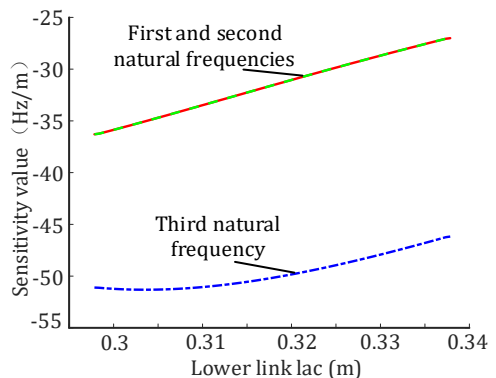
a) Sensitivity of natural frequency to l_{bc}



b) Sensitivity of natural frequency to r_b



c) Sensitivity of natural frequency to θ



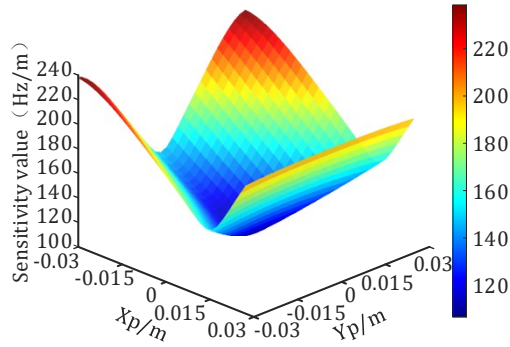
d) Sensitivity of natural frequency to l_{ac}

Figure 5: Sensitivity of natural frequency to various structural parameters

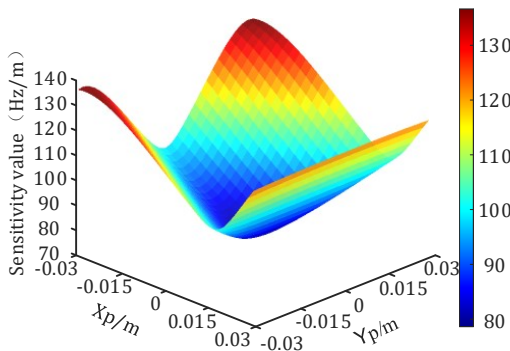
It can be seen from Fig. 5a) and Fig. 5b) that the sensitivity of each natural frequency shows an increasing trend, indicating that each natural frequency gradually increases with the increase of l_{bc} and r_b . It can be seen from Fig. 5c) and Fig. 5d) that the sensitivity of each natural frequency of the mechanism is negative, and the absolute values of the sensitivities gradually decrease, indicating that each natural frequency gradually decreases with the increase of θ and l_{ac} .

The third natural frequency of the mechanism is relatively high. According to equation (8), the pose variation diagrams of the sensitivity of the third natural frequency to each structural parameter

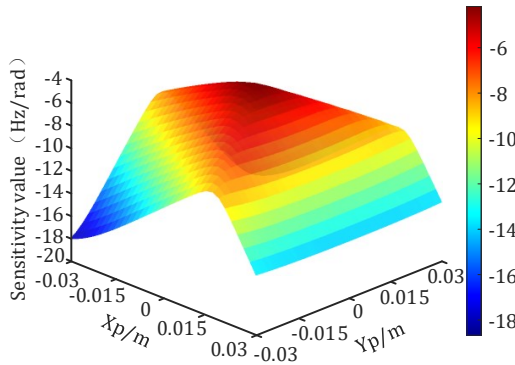
within the initial height plane of the mechanism are obtained, as shown in Fig. 6.



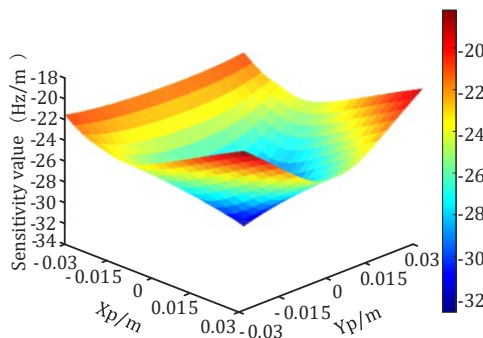
a) Sensitivity of the natural frequency to l_{bc}



b) Sensitivity of the third natural frequency to r_b



c) Sensitivity of the third natural frequency to θ



d) Sensitivity of the third natural frequency to l_{ac}
Figure 6: Sensitivity of the third natural frequency to each parameter

As illustrated in Fig. 6, the mechanism shows significantly higher sensitivity to the parameters l_{bc} and r_b than to l_{ac} and θ . This indicates that changes in l_{bc} and r_b exert a more pronounced influence on the natural frequency. Therefore, when the natural frequency of the mechanism fails to meet the practical operational requirements, priority should be given to optimizing and adjusting the structural dimensions corresponding to these two key parameters, so as to effectively regulate the natural frequency and improve the vibration performance of the mechanism.

4. Dimensional Optimization of the Orientation-Adjusting and Vibration-Isolation Platform

4.1 Optimization Performance Index

The acceleration term $\ddot{\mathbf{q}} = \mathbf{G}_q^p \mathbf{A}_{op} - \dot{\mathbf{q}}^T \mathbf{H}_q^p \dot{\mathbf{q}}$ plays a dominant role in the orientation-adjusting and vibration-isolation platform [18]. The dynamic model is rearranged and simplified as follows

$$\boldsymbol{\tau}_A = \mathbf{E} \mathbf{A}_{op} \quad (9)$$

where, $\mathbf{E} = \mathbf{M}_q^T \mathbf{G}_p^q$. On the basis of the simplified dynamic model, given that the moving platform produces a unit acceleration, the maximum eigenvalue of the mass matrix $\mathbf{E}^T \mathbf{E}$ is solved to define the dynamic performance index [19]

$$\sigma_1 = \sqrt{\eta_{\max}} \quad (10)$$

The dynamic performance index reflects the magnitude of the inertial force of the moving platform mapped to the generalized driving joint forces. For the passive vibration isolation mode of the orientation-adjusting and vibration-isolation platform, a smaller value of this index indicates a smaller force mapped to the generalized driving joints, less disturbance to the springs, a more stable working point of the springs, and higher linearity and vibration isolation purity of the system.

The natural frequency determines the starting position of the effective vibration isolation region. The lower the natural frequency, the lower the starting frequency corresponding to this critical point. The third natural frequency of the orientation-adjusting and vibration-isolation platform is significantly higher than the first and second natural frequencies. Therefore, the maximum value of the third natural frequency is adopted as the evaluation index, that is

$$\sigma_2 = \omega_{n3\max} \quad (11)$$

4.2 Dimensional Optimization

The weighted sum method is a commonly used quantitative approach in multi-criteria decision-making and comprehensive evaluation. A single-objective multivariable optimization for a single performance index is implemented using a genetic algorithm, with each objective function expressed as [20]

$$f_i(x) = \frac{\sigma_i(x) - \sigma_{\min}(x)}{\sigma_{\max}(x) - \sigma_{\min}(x)} \quad (22)$$

In the formula, $\sigma_{\max}(x)$ and $\sigma_{\min}(x)$ are the maximum and minimum values of each performance index within the variation range of the key parameters, respectively.

Weights characterize the relative importance of individual performance indicators and exert a direct influence on the rationality of evaluation results, which are formulated as

$$\sigma_{\min}(x) = \sum_{i=1}^2 \omega_i f_i(x) \quad (13)$$

In the formula, ω_i is the weight coefficient, and $f_i(x)$ represents each performance index. In this study, the weight coefficients for dynamic performance and natural frequency are both set to 0.5.

Using the parameters listed in Table 1 as initial parameters and specifying the optimization ranges of the structural parameters, the objective function of equation (13) is optimized via a genetic algorithm. The population size is set to 100, the roulette wheel selection method is adopted, the crossover probability is 0.8, and the adaptive mutation method is selected. The termination generation is set to 100. The optimization results are shown in Fig. 7 and Table 2.

It can be seen from Fig. 7 that convergence is approached after the 70th generation, at which point the minimum value of the comprehensive optimization objective function of the mechanism is $f_{\min}(x) = 0.361$.

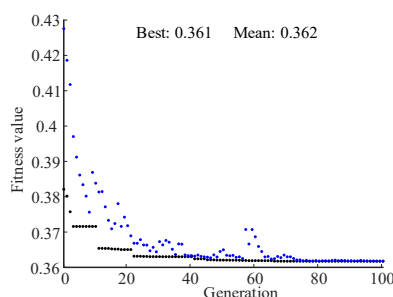
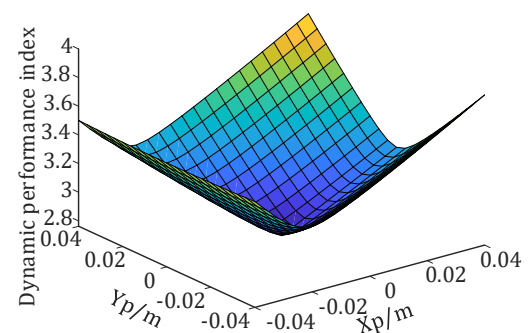


Figure 7: Comprehensive optimization results

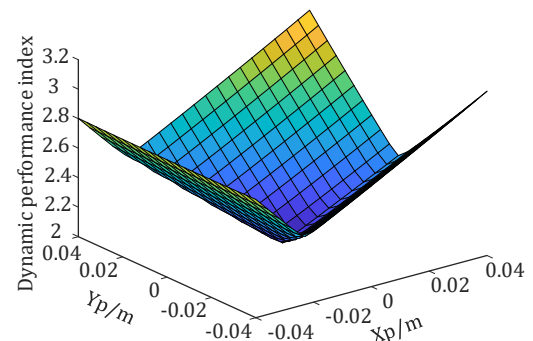
Table 2. Comprehensive optimization results

Parameter	Optimization range	Optimization results
l_{bc}/m	[0.30,0.35]	0.30
r_b/m	[0.15,0.20]	0.17
$\theta/^\circ$	[45,65]	63
l_{ac}/m	[0.30,0.35]	0.34

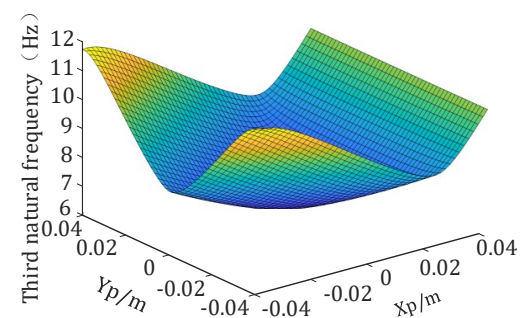
Based on the dimensions before and after optimization, the global performance surfaces of the orientation-adjusting and Vibration-isolation platform are obtained, as shown in Fig. 8 and 9.



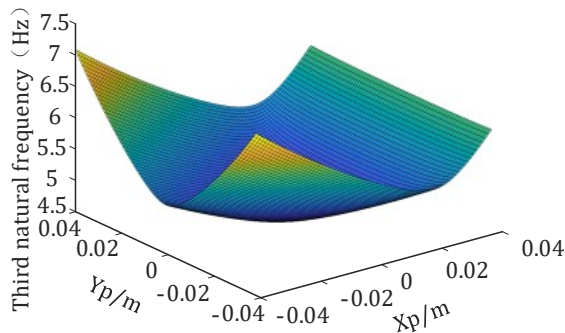
a) Dynamic performance index before optimization



b) Dynamic performance index after optimization
Figure 8: Dynamic performance surfaces before and after optimization



a) Natural frequency before optimization



b) Natural frequency after optimization
Figure 9: Natural frequency surfaces before and after optimization

It can be seen from the figure that both the dynamic performance and the natural frequency are improved after optimization, with the dynamic performance increased by approximately 20% and the natural frequency decreased by about 40%.

5. Conclusions

Based on the work of Ref. [10], this paper analyzes the global variation law of natural frequencies with respect to pose, establishes a sensitivity model of natural frequencies to key structural parameters, and clarifies the degree of influence of each structural parameter and their primary-secondary relationships.

To address the inherent conflict between pose-adjustment accuracy and vibration-isolation bandwidth, the dynamic performance index and the natural frequency are taken as optimization objectives, key structural parameters are selected as optimization variables, and a multi-objective optimization is carried out using a genetic algorithm combined with the weighted sum method. After optimization, the dynamic performance is improved by approximately 20%, while the natural frequency of the system is reduced by about 40%, effectively broadening the passive vibration isolation bandwidth. The overall performance is significantly superior to the empirically selected parameter scheme, providing a quantitative basis for the engineering design of similar hybrid orientation-adjusting and vibration-isolation platforms.

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