

OPTIMAL PERFORMANCE OF A VIBRATION ABSORBER USING HARMONIC ANALYSIS

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Abstract - This paper presents a damped dynamic absorber system with an optimal design. In order to build the dynamic absorber for improved performance, the principles of negative stiffness and mass are introduced and discussed in depth together with their physical justification. To meet optimization constraints and acquire the best values for design parameters, the harmonic analysis is examined and the harmonic response for an external excitation is analytically determined. The optimal damping ratios and frequency ratios for six mass ratios scenarios are simultaneously determined. The findings indicate that damping ratios are crucial variables, even for minor uncertainties. The nonlinear harmonic response was altered by dampening. The best values for ideal damping ratios and ratio of frequency must be carefully chosen because they are not independent. Zero response is achieved for the amplitude response when (< 0.25). The frequency ratio affects the response only in the range of ($0.5 < < 1.5$).

Keywords: Dynamic absorber, Optimization, negative stiffness, Negative mass.

1. Introduction

Dynamic Vibration Absorbers (DVA) are one of the most widely used devices as vibration control devices due to their efficiency, dependability, and affordability, as Frahm [1][2] developed the first DVA without damping in 1909. A spring and mass are traditionally attached to a vibrating body that vibrates harmonically or in a narrowband frequency range as an undamped DVA. Numerous works on optimization design [3–7] and vibration absorber tuning [8]–[12] have been done. Some of the above researches are basically based on positive springs effect. Another concept introduced is the negative stiffness, which denotes the reversal of the relationship between external force and displacement in deformed objects. The works in [13]– [19] reported on the characteristics and stability requirements of the negative stiffness system. Al-Shudeifat and Butcher [20] developed a comprehensive nonlinear analytical model to accurately represent absorber behavior under large-

amplitude excitations, emphasizing the importance of advanced modeling approaches in specific operating regimes. In industrial and manufacturing engineering, vibrations generated by heavy machinery, generators, and diesel engines can threaten structural integrity, especially when these devices are mounted on steel frameworks or floors [21]. The system with negative stiffness (NS) has a higher load-bearing capacity than the system with only positive stiffness, and its natural frequency will also be decreased. By using the buckling of beams under axial load, [22] produced NS around equilibrium and obtained an isolation system by fusing it with a linear positive spring. The NS principle was also used for vibration isolation (VIS) as reported by [23] [24] who put forth a new VIS in which an active control technique was used to achieve NS. In [25], a passive DVA as used with an analytical and experimental NS mechanism. A nonlinear vibration isolator (NVI) with a negative stiffness mechanism was also studied by [26]– [28]. The available analytical solutions for optimal

parameters are time costly, limited in application, and not valid when design parameters that vary in values [29]. There has been little written about the ideal DVA with NS and negative mass (NM) parameters in detail, despite the fact that the use of NS in vibration isolation systems has been the subject of extensive research. In this work, the principles of negative mass and stiffness are explained along with their physical concepts. The optimal DVA parameters are determined using the direct research method, by adding a new constraint to the main objectives. The frequency ranges of the optimal values are studied as well for six cases of different mass ratios.

2. Metamaterial Negative Properties

2.1 Negative Effective Mass

Fig. 1 shows a 2-DOF system exposed to a harmonic excitation. The frequency response functions (FRFs) H_{11} and H_{22} between the response $u_1(t)$ and $u_2(t)$ and the input harmonic force $F(t)$, are given by [7][9]:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F(t) \\ 0 \end{Bmatrix}$$

$$F \equiv F_0 e^{j\Omega t}, \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \equiv \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} e^{j\Omega t}$$

$$\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \equiv \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} e^{j\Omega t} \quad (1)$$

$$\text{if } n=1, H_{11} \equiv \frac{a_1}{F_0} = \frac{k_2 - m_2 \Omega^2}{(k_2 - m_1 \Omega^2)(k_2 - m_2 \Omega^2) - k_2^2}$$

$$\text{if } n=2, H_{21} \equiv \frac{a_2}{F_0} = \frac{k_2}{(k_2 - m_1 \Omega^2)(k_2 - m_2 \Omega^2) - k_2^2}$$

For more convenience, the following terms are simplified as:

$$\tilde{m}_1 \equiv \frac{F}{u_1} = \frac{F_0}{-\Omega^2 a_1} = m_1 + \frac{m_2}{1 - \frac{\Omega^2}{\omega_2^2}}, \omega_2 \equiv \sqrt{\frac{k_2}{m_2}} \quad (2)$$

Where m_1 and m_2 are the principal and absorber masses, respectively. F is the excitation harmonic forces applied at the principal mass. u_1 and u_2 are the degrees of freedom of principal and absorber mass, receptively. The symbol Ω is the excitation frequency, $j \equiv \sqrt{-1}$, $\dot{u}_1 \equiv du_1/dt$, t is time, and ω_2 is the resonance frequency of the added absorber representing by the stiffness k_2 and mass m_2 . The effective mass $\tilde{m}_1$ is calculating by considering the presented system in Fig. 1 as a single degree of freedom instead of two degrees of freedom. If $\Omega = \omega_2$, Eqn. (1) shows that $|\tilde{m}_1|$ is ∞ , and

$$H_{11} = u_1(t) = 0, F_0 = -k_2 a_2 \quad (3)$$

$$F(t) = -k_2 u_2(t) = m_2 \ddot{u}_2(t) \quad (4)$$

This means that the inertia force of the mass of absorber ($-m_2 \ddot{u}_2$) has cancelled the effect of the external force by the presence of the spring K , which is considered as a good point to start to imply the time response of the principal mass $u_1(t)=0$. The zero response of this mass is the main key of vibration reduction concept by using the vibration absorber.

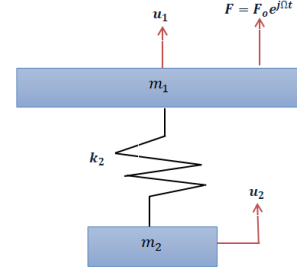


Figure 1: Mass in mass 2-DOF system

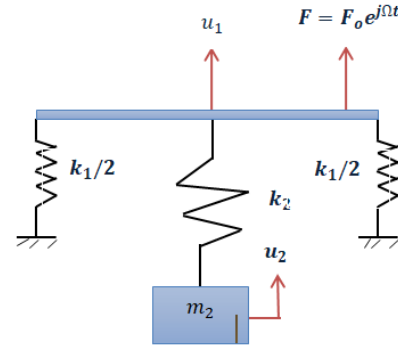


Figure 2: Spring-mass system with effective stiffness

2.2 Negative Effective Stiffness

Considering the 2D-O-F system illustrated in Fig. 2, this system is similar to Fig. 1, but another spring K_1 is added, forming two sub springs at the left and right ends of the principal mass. The EOMs and the corresponding magnification factors $H_{n1}(\Omega)$ ($n=1,2$) are given by:

$$\begin{bmatrix} 0 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \end{Bmatrix}$$

$$F \equiv F_0 e^{j\Omega t}, \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \equiv \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} e^{j\Omega t} \quad (5)$$

$$\text{if } n=1, H_{n1} = \frac{a_1}{F_0} = \frac{k_2 - m_2 \Omega^2}{(k_1 + k_2)(k_2 - m_2 \Omega^2) - k_2^2} = \frac{1}{k_1}$$

$$\text{if } n=2, H_{n1} = \frac{a_2}{F_0} = \frac{k_2}{(k_1 + k_2)(k_2 - m_2 \Omega^2) - k_2^2}$$

For more convenience, the following terms are simplified as:

$$\tilde{k}_1 \equiv \frac{F}{u_1} = \frac{F_0}{a_1} = k_1 + \frac{k_2}{1 - \frac{\Omega^2}{\omega_2^2}}, \omega_2 \equiv \sqrt{\frac{k_2}{m_2}} \quad (6)$$

where, $\tilde{k}_1$ is the effective spring when the presented system is considered as a 1-DOF instead of 2-DOF. If $\Omega = \omega_2$, "(6)" shows that $|\tilde{k}_1| \rightarrow \infty$ and

$$H_{11} = u_1 = 0 \quad (7)$$

$$F(t) = -k_2 u_2(t) = m_2 \ddot{u}_2(t) \quad (8)$$

This means that the mass of absorber ($-m_2 \ddot{u}_2$) has cancelled the effect of the external force by the presence of the spring k_2 , and hence, $u_1(t) = 0$. This also discloses that when $\Omega = \omega_2$, the term a_2 increase when m_2 decreases. More importantly is that when the $k_2 / (\omega_2^2 / \Omega^2 - 1) > k_1$ and $\Omega < \omega_2$, the effective value of the stiffness $\tilde{k}_1$ becomes negative (NS). It is important to explain that when $\tilde{k}_1 < 0$ and $\Omega < \omega_2$, a developed downward force is built up in the spring and pulled against $F(t)$, and it can be given by:

$$k_2(a_1 - a_2) = F_0(\tilde{k}_1 - k_1) / \tilde{k}_1 \quad (9)$$

The developed force is greater than the force according to "(9)". This explains the negative value of the stiffness

2.3 Design of a Spring-mass Absorber for Vibration Attenuation

The governing differential equations of the physical system described in Fig. 3 can be determined using Newton's second law as follow:

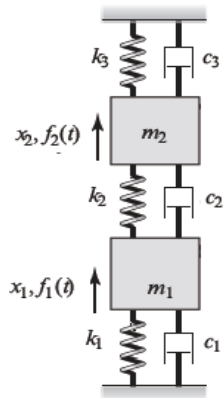


Figure 3: Damped 2-DOF vibratory system

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 + c_3 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} f_1(t) \\ f_2(t) \end{Bmatrix} \quad (10)$$

where $m_1, m_2, m_3, c_1, c_2, c_3, k_1, k_2, k_3$ are the system masses, dampers, and stiffness, respectively.

$f_1(t)$ and $f_2(t)$ are the loads applied on the first and second masses, respectively. The force vector is given by:

$$f_1(t) = F_1 e^{j\omega t}, f_2(t) = F_2 e^{j\omega t} \quad (11)$$

The general solution of Eq. 3.45 can be presumed as:

$$x_i(t) = X_i(j\omega) e^{j\omega t} \quad (12)$$

Where is the excitation frequency. The general solution ω of "(10)" can be presumed as:

$$\begin{bmatrix} a_{11}(j\omega) & a_{12}(j\omega) \\ a_{21}(j\omega) & a_{22}(j\omega) \end{bmatrix} \begin{Bmatrix} X_1(j\omega) \\ X_2(j\omega) \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \quad (13)$$

Eq. (13) can be solved for obtaining the amplitude of the presumed time response:

$$X_1(j\omega) = \frac{a_{22}(j\omega)F_1 - a_{12}(j\omega)F_2}{D_o(j\omega)} \quad (14)$$

$$X_2(j\omega) = \frac{a_{11}(j\omega)F_2 - a_{21}(j\omega)F_1}{D_o(j\omega)}$$

Where:

$$\begin{aligned} D_o(j\omega) &= a_{11}(j\omega)a_{22}(j\omega) - \\ &a_{12}(j\omega)a_{21}(j\omega) \end{aligned} \quad (15)$$

To study the vibration effect, the force F_2 is set to be zero ($F_2=0$). Thus, "(14)" can be updated as:

$$\lambda M_{11}(j\omega) = \lambda \frac{X_1(j\omega)}{F_1} = -\lambda \frac{a_{22}(j\omega)}{D_o(j\omega)} \quad (16)$$

$$\lambda M_{21}(j\omega) = \lambda \frac{X_2(j\omega)}{F_1} = -\lambda \frac{a_{21}(j\omega)}{D_o(j\omega)}$$

where $\lambda M_{11}(j\omega)$ and $\lambda M_{21}(j\omega)$ are frequency-response functions due to the effect of F_1 only.

The parameter λ measured by (N/m) is a dimensional parameter introduced to make "(17)" non-dimensional. The general form of the frequency response can be given by:

$$H_{ij}(\omega) = \beta |M_{ij}(j\omega)| \quad i, j = 1, 2 \quad (17)$$

Back to the system presented in Figure 3 above, the matrix $a_{ij}(j\omega)$ given in "(13)", with some non-dimensional parameters, can be determined as follows:

$$\begin{aligned} a_{11}(j\omega) &= A(j\Omega)k_1, \\ a_{12}(j\omega) &= -B(j\Omega)B(j\Omega)k_1 m_r, \\ a_{21}(j\omega) &= -k_1 m_r, \\ a_{22}(j\omega) &= D(j\Omega)k_1 m_r, \end{aligned} \quad (18)$$

where:

$$\begin{aligned} A(j\Omega) &= -\Omega^2 + 2(\zeta_1 + \zeta_2 m_r \omega_r) j\Omega + 1 + m_r \omega_r^2 \\ B(j\Omega) &= 2\zeta_2 \omega_r j\Omega + \omega_r^2 \\ D(j\Omega) &= -\Omega^2 + 2\zeta_2 \omega_r (1 + c_{32}) j\Omega + \omega_r^2 (1 + k_{32}) \end{aligned} \quad (19)$$

and,

$$\Omega = \frac{\omega}{\omega_{n1}}, \omega_r = \frac{\omega_{n2}}{\omega_{n1}}, c_{32} = \frac{c_3}{c_2}, 2\zeta_i = \frac{c_i}{m_i \omega_{ni}} (i = 1, 2)$$

Based on this simplification, “(16)” can be simplified to $D_0(j\omega) = k_1^2 m_r R(j\Omega)$ where $R(j\Omega)$ can be expressed by:

$$\begin{aligned} R(j\Omega) &= \Omega^4 - 2j\Omega^3 (\zeta_2 \omega_r (c_{32} + m_r + 1) + \zeta_1) - \\ &\Omega^2 (c_{32}^2 (4\zeta_2^2 m_r \omega_r^2 + 4\zeta_1 \zeta_2 \omega_r) + k_{32} \omega_r^2 + m_r \omega_r^2 + \\ &4\zeta_1 \zeta_2 \omega_r + \omega_r^2 + 1) + k_{32} m_r \omega_r^4 k_{32} \omega_r^2 + \omega_r^2 + \\ &2j\Omega \omega_r \left(c_{32} (\zeta_2 + \zeta_2 m_r \omega_r^2) + \zeta_1 \omega_r + \right. \\ &\left. \zeta_2 + k_{32} (\zeta_2 m_r \omega_r^2 + \zeta_1 \omega_r) \right) \end{aligned} \quad (20)$$

Where $k_{32} = k_3 / k_2$

Finally, the frequency response functions described in “(17)” can be restated as follows:

$$\begin{aligned} H_{11}(\omega) &= k_1 \left| \frac{k_1 m_r D(j\Omega)}{k_1^2 m_r R(j\Omega)} \right| = \left| \frac{D(j\Omega)}{R(j\Omega)} \right| \\ H_{21}(\omega) &= k_1 \left| \frac{-k_1 m_r B(j\Omega)}{k_1^2 m_r R(j\Omega)} \right| = \left| \frac{B(j\Omega)}{R(j\Omega)} \right| \\ H_{12}(\omega) &= k_1 \left| \frac{-k_1 m_r B(j\Omega)}{k_1^2 m_r R(j\Omega)} \right| = \left| \frac{B(j\Omega)}{R(j\Omega)} \right| \\ H_{22}(\omega) &= k_1 \left| \frac{k_1 A(j\Omega)}{k_1^2 m_r R(j\Omega)} \right| = \left| \frac{A(j\Omega)}{m_r R(j\Omega)} \right| \end{aligned} \quad (21)$$

3. Results and Discussion

The main objective of this work is to investigate the effect of the more sensitive parameters of a spring-mass absorber, and obtain the optimal values of a unit cell absorber (optimal spring and optimal mass for the absorber) for enhancing the absorption capacity and vibration attenuation. The principles of negative mass and negative stiffness are discussed in detail.

3.1 Negative Mass

Figure 4 shows the variations of effective mass with frequency ratio (Ω/ω_2) for different values of mass absorber $m_2 = (0.1 - 0.7)m_1$. In general, effective mass varies with the excitation frequency Ω and absorber mass m_2 . When $\Omega > \omega_2$, the effective mass denoted by Eqn. (3.16) becomes negative (negative mass). This character is the key

point of using metamaterials for vibration attenuation that corresponds to $u_1(t) = 0$, as shown by Eqn. 3.17. However, when $\Omega < \omega_2$, the effective mass is positive and hence the two terms $\frac{F_0}{a_1} = -\tilde{m}_1 \Omega^2$ and $\frac{F_0}{a_2}$ are both negative values which means that no one of $u_1(t)$ and $u_2(t)$ equals to zero and also means that both of the masses in the system vibrate in same phase. Hence, vibration of the main mass cannot be attenuated. In the other hand, Figure 4 shows that the absolute value of the effective mass increase with increasing absorber mass. However, this increment become less sensitive at higher values of the absorber mass.

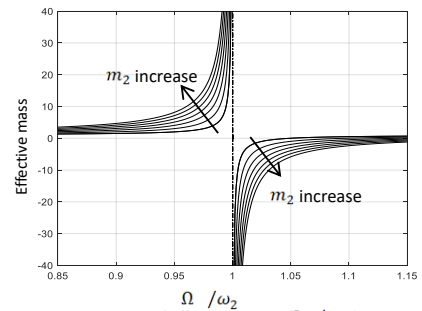


Figure 4: Variations of effective mass (Ω / ω_2), $m_1 = 1 \text{ kg}$, $m_2 = (0.1 - 0.7)m_1$.

3.2 Negative Stiffness

Figure (5) shows the variations of effective spring with frequency ratio (ω_2 / Ω) for different values of absorber spring $k_2 = (0.1 - 0.7)k_1$. The variation of the effective spring due variations of the excitation frequency is similar to that corresponding of the effective mass. However, the frequency ratio is represented by ω_2 / Ω in Fig. 5.2 instead of the Ω/ω_2 for the negative mass represented by Figure 5. In other words, $\omega_2 > \Omega$, the effective spring denoted by Eqn. 6 is negative. A developed downward force built up in the spring pulled against the applied force $F(t)$. The developed force is generally greater than F_0 according to Eqn. 9. This behavior explains the importance of the negative stiffness value for vibration attenuation.

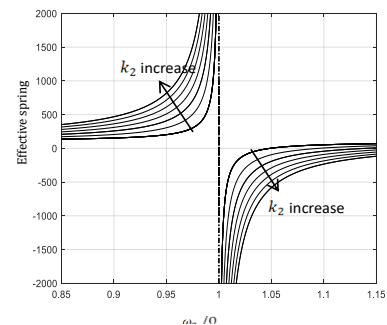


Figure 5: Variations of effective spring, $k_1 = 100 \text{ N/m}$, $k_2 = (0.1 - 0.7)k_1 \text{ N/m}$.

3.3 Application of 2 DOF System as Vibration Absorber

Two designs of absorbers are investigated. The sensitivity of the absorber performance to the variations of the design parameters is studied as well. The primary and secondary systems are assumed to be damped by introducing ζ_1 and ζ_2 for both systems, respectively. However, to reduce the design requirements, let us first assume that $\zeta_2 = 0$. The key point of designing a good vibration absorber is the selection of proper values for m_2 and k_2 to reduce response of the main vibratory system. The only way to get zero responses for the primary mass is by making the terms $a_{22}(j\omega)$ or $a_{22}(j\omega)$ or $D(j\omega)$ equal to zero when $\Omega = \omega_r$.

3.4 Effect of the Damping Factor ζ_1 on the Dynamic Response

The effect of the first damping ratio is presented in Figure 6 for three values of the frequency ratio, while the mass ratio is held constant. In general, no infinite amplitude response is presented because both introduced damping ratios are not equal to zero. The amplitude response decreases dramatically as the damping ratio increase. The responses are very close at the excitation frequency, being smaller and greater than the first and second resonance frequencies, respectively. However, the behavior is different as the frequency ratio increase. Increasing the damping ratio from (0.1 to 0.4) reduced the amplitude response by 400%.

3.5 Effect of the Damping Factor ζ_2 on the Dynamic Response

Figure 7 shows the variation of amplitude responses due to the variation of ζ_2 for three values of frequency ratios. It is noticed that the peaks disappear as the damping ration increase. Even beak locations change as the frequency ratio increase. The greater effect of ζ_2 is noticed at its maximum value and when $\Omega = 1$. Figure 6 and Figure. 7 show that ζ_1 is more dominant on the amplitude response than ζ_2 . However, the response of the primary mass can be controlled by choosing suitable values of ζ_2 .

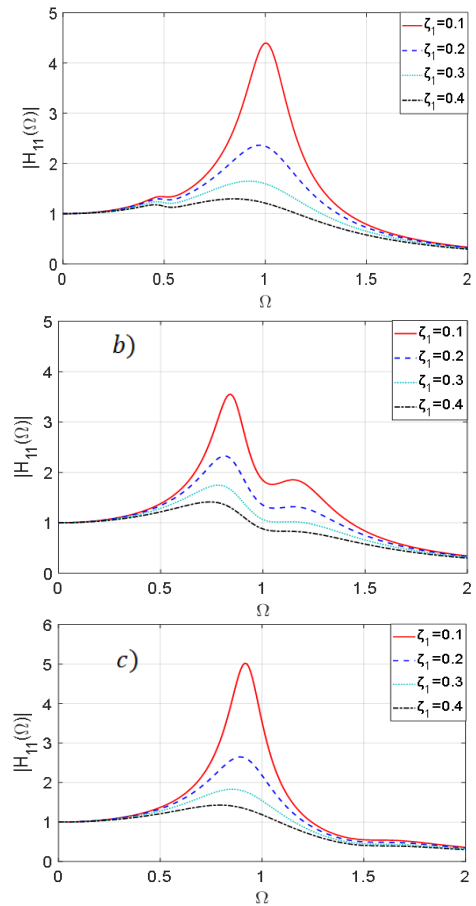


Figure 6: Amplitude response when $\zeta_2 = 0.2$, $m_r = 0.1$
a) $\omega_r = 0.5$, b) $\omega_r = 1$, c) $\omega_r = 1.5$

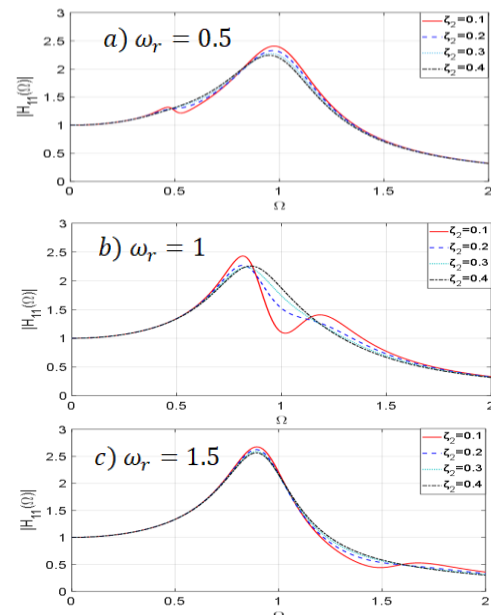


Figure 7: Amplitude response when $\zeta_1 = 0.2$, $m_r = 0.1$
a) $\omega_r = 0.5$, b) $\omega_r = 1$, c) $\omega_r = 1.5$

3.5 Effect of Mass Ratio (m_r) on the Dynamic Response

The effect of the (m_r) is presented in Figure 8. The smaller value of the mass ratio altered the amplitude response to be smoother, and the peaks gradually disappeared. However, the higher values of the mass ratio peaks become more obvious for higher amplitudes, specifically when the frequency ratio is equal to or more than 1. It is worth mentioning that the variation of frequency and mass ratio show no effect on the amplitude response when ($\Omega < 0.25$).

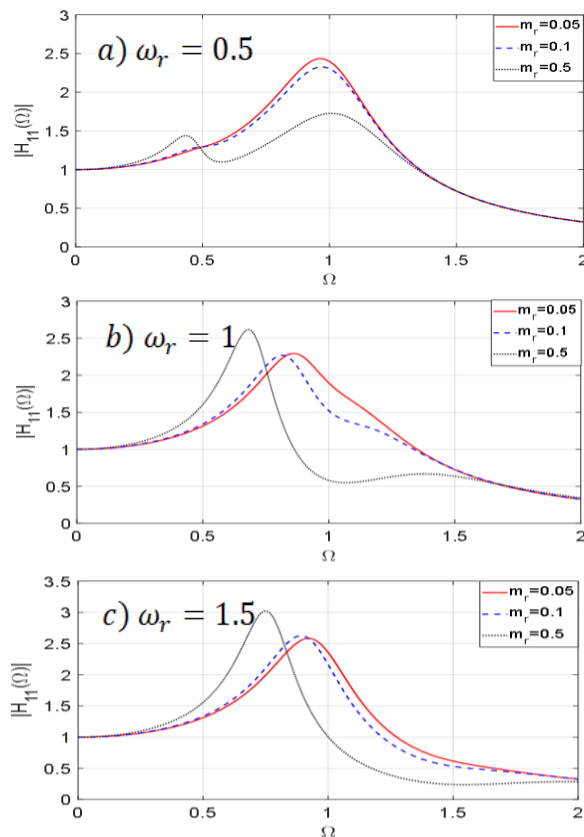


Figure 8: Amplitude response when $\zeta_1 = \zeta_2 = 0.2$,
a) $\omega_r = 0.5$, b) $\omega_r = 1$, c) $\omega_r = 1.5$

3.6 Effect of Frequency Ratio on the Dynamic Response

Figure 9 shows the effect of the frequency ratio on the amplitude response for different values of mass ratios. The range of the non-dimensional frequency Ω at which the amplitude response showed change in between is ($0.5 < \Omega < 1.5$). The amplitude responses are invariant for the other values. In addition, the difference in behavior is smaller at the smaller values of the frequency ratios, and it increases dramatically as this ratio to 1.5.

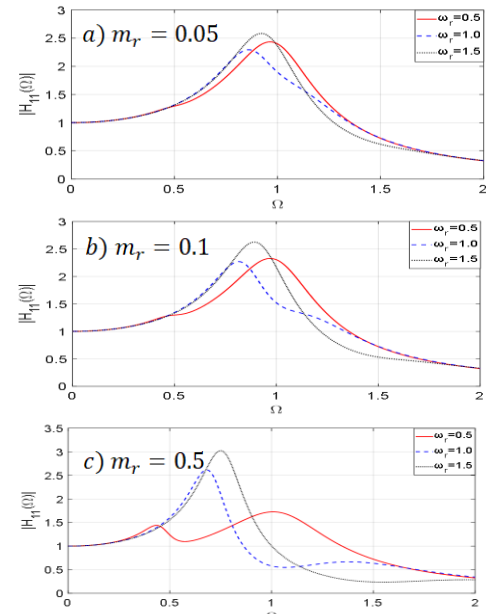


Figure 9: Amplitude response when
a) $m_r = 0.05$, b) $m_r = 0.1$, c) $m_r = 0.5$

3.7 Optimal Design

The response of the primary mass can be attained to be zero when $\zeta_2 = 0$. However, this condition is ideal because of the existence of the structural and frictional damping. In this case, the selection of the optimal combinations of system parameters is not an easy task because of the nonlinear and fluctuated behavior of ζ_2 on the absorber performance. Figure 10 shows the effect of ζ_2 on the amplitude response in detail. Similar behavior is verified in previous sections. The response is zero when $\zeta_2 = 0$. The amplitude response of the peaks decreases, whereas both peaks ($\Omega \neq 0$) go to infinite when $\zeta_2 = 0$ and $\zeta_2 = \infty$.

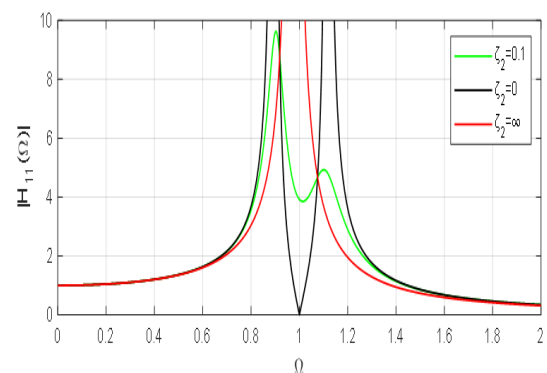


Figure 10: Nonlinear effect of damping at $\zeta_1 = 0.1$, $m_r = 0.1$, $w_r = 1$

In other words, there is a critical value of ζ_2 in between these values at which the peaks are smaller or minimal.

For optimization purposes, the best behavior for these curves are the curves which make the difference between the two peaks in frequency response as small as possible for a wide range of non-dimensional frequencies, including $\Omega = 1$ when $\zeta_2 \neq 0$ [22]. In this work, another constraint is added to get a smaller response of the primary mass. Therefore, six cases are studied for six values of m_r . The direct research method is used to obtain optimal values, as presented in Table I. These optimal values are determined at $\zeta_1 = 0.1$. Figure 11 shows several cases of response.

Table 1. Optimal values of absorber parameters for six cases (m_r).

m_r	$\zeta_2(N.s/m)$	ω_r	Peaks difference	Frequency range
0.1	0.195	0.859	$3.55 \cdot 10^{-3}$	0.789 to 1.051
0.2	0.259	0.771	$2.74 \cdot 10^{-3}$	0.716 to 1.031
0.3	0.301	0.696	$3.84 \cdot 10^{-3}$	0.647 to 1.015
0.4	0.351	0.649	$9.55 \cdot 10^{-3}$	0.303 to 0.965
0.5	0.407	0.603	$3.52 \cdot 10^{-3}$	0.586 to 0.920
0.6	0.321	0.574	$3.83 \cdot 10^{-3}$	0.510 to 1.022

In other words, once the mass ratio gets bigger, the frequency of the maximum peaks shifts to the left, and the frequency range varies and may not cover the resonance frequency as noticed when $m_r = 0.4$ and 0.5. This limits the performance of the DVA.

The optimal response is the dashed curves, where the differences between peaks are the smallest for each subfigure. These figures show that the minimum amplitude for the primary system is attained at the higher value of mass ratio. However, this corresponds to a limitation in the frequency range.

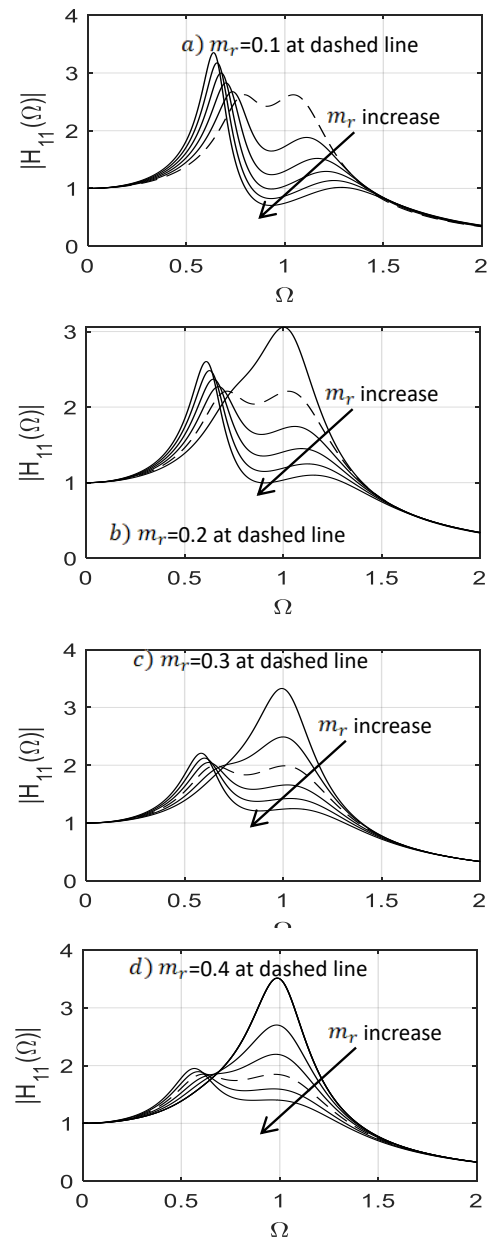


Figure 11: Optimal values for absorber parameters for different values of mass ratio m_r .

It is important to notice that while the optimal values of the parameters presented in Table 1 count based on the theoretical design concepts, the practical design and implementations must consider the manufacturing tolerances; linear and nonlinear behavior of the spring, temperature-dependent damper viscosity and other uncertainties. Sensitivity analysis and robustness analysis under these tolerances remains critical topic for future experimental studies.

4. Conclusions

The principles of negative mass and negative springs have been studied in details, and they are used for designing a dynamic absorber. The results of this work showed that damping ratios and frequency ratio are critical parameters for designing a successful dynamic absorber. Zero response is achieved for the amplitude response when ($\Omega < 0.25$). The frequency ratio affects the response only in the range of ($0.5 < \Omega < 1.5$). Finally, the optimal design can be attained by selecting optimal values for ζ_2 and ω_r . Every system frequency requires specific spring-mass values of better attenuation.

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References

- [1] Frahm, H. (1909). Device for damping vibrations of bodies (U.S. Patent No. 0989958). U.S. Patent and Trademark Office.
- [2] Frahm, H. (1911). Device for damping vibrations of bodies (U.S. Patent No. 989,958). U.S. Patent and Trademark Office.
- [3] Snowdon, J. C., & Jones, D. I. G. (1969). Vibration and shock in damped mechanical systems. *Journal of Applied Mechanics*, 36(2), 338–350.
- [4] Randall, S. E., Halsted, D. M., III, & Taylor, D. L. (1981). Optimum vibration absorbers for linear damped systems. *Journal of Mechanical Design*, 103(4), 908–913.
- [5] Lei, Z., & Nayfeh, S. A. (2004). Minimax optimization of multi-degree-of-freedom tuned-mass dampers. *Journal of Sound and Vibration*, 272(3–5), 893–908.
- [6] Fortgang, J., & Singhose, W. (2005). Design of vibration absorbers for step motion and step disturbances. *Journal of Mechanical Design*, 127(1), 160–163.
- [7] Liu, K., & Coppola, G. (2010). Optimal design of damped dynamic vibration absorber for damped primary systems. *Transactions of the Canadian Society for Mechanical Engineering*, 34(4), 119–135.
- [8] Liu, K., & Liu, J. (2005). The damped dynamic vibration absorbers: Revisited and new result. *Journal of Sound and Vibration*, 248(3–5), 1181–1189.
- [9] Zuo, L., & Nayfeh, S. A. (2006). The two-degree-of-freedom tuned-mass damper for suppression of single-mode vibration under random and harmonic excitation. *Journal of Vibration and Acoustics*, 128(1), 56–65.
- [10] Ghosh, A., & Basu, B. (2007). A closed-form optimal tuning criterion for TMD in damped structures. *Structural Control and Health Monitoring*, 14(4), 681–692.
- [11] Wong, W., & Cheung, Y. (2008). Optimal design of a damped dynamic vibration absorber for vibration control of structure excited by ground motion. *Engineering Structures*, 30(1), 282–286.
- [12] Viana, F., Kotinda, G., & Rade, D. A. (2008). Tuning dynamic vibration absorbers by using ant colony optimization. *Computers & Structures*, 86(13–14), 1539–1549.
- [13] Lakes, R. S. (2001). Extreme damping in composite materials with a negative stiffness phase. *Physical Review Letters*, 86(13), 2897–2900. (Note: Fixed typo "Latkes" to "Lakes")
- [14] Lakes, R. S. (2001). Extreme damping in compliant composites with a negative-stiffness phase. *Philosophical Magazine Letters*, 81(2), 95–100.
- [15] Lakes, R. S., Lee, T., Bersie, A., & Wang, Y. C. (2001). Extreme damping in composite materials with negative-stiffness inclusions. *Nature*, 410(6828), 565–567. (Note: Expanded "et al." to standard APA multi-author format)
- [16] Lakes, R. S., & Drugan, W. J. (2002). Dramatically stiffer elastic composite materials due to a negative stiffness phase? *Journal of the Mechanics and Physics of Solids*, 50(5), 979–1009.
- [17] Wang, Y. C., & Lakes, R. S. (2004). Extreme stiffness systems due to negative stiffness elements. *American Journal of Physics*, 72(1), 40–50.
- [18] Wang, Y. C., & Lakes, R. (2004). Negative stiffness-induced extreme viscoelastic mechanical properties: Stability and dynamics. *Philosophical Magazine*, 84(35), 3785–3801.
- [19] Wang, Y. C., & Lakes, R. (2005). Stability of negative stiffness viscoelastic systems. *Quarterly of Applied Mathematics*, 63(1), 34–55.
- [20] Askes, H., Lombardo, M., & Nguyen, D. C. D. (2024). Homogenisation of periodic lattices with lumped and distributed mass: Beam models, continualisation and stabilisation. *International Journal of Solids and Structures*, 302, Article 112988.
- [21] Li, F., Wu, H., Liu, L., Ye, Y., Wang, Y., & Wu, P. (2025). Nonlinear optimal frequency control for dynamic vibration absorber and its application. *Mechanical Systems and Signal Processing*, 223, Article 111932.
- [22] Platus, D. L. (1992). Negative-stiffness-mechanism vibration isolation systems. In *Vibration Control in Microelectronics, Optics, and Metrology* (Vol. 1619, pp. 44–54). SPIE.

- [23] Mizuno, T., Toumiya, T., & Takasaki, M. (2003). Vibration isolation system using negative stiffness. *JSME International Journal Series C Mechanical Systems, Machine Elements and Manufacturing*, 46(3), 807–812.
- [24] Huang, X., Su, Z., & Hua, H. (2019). Optimal parameters for dynamic vibration absorber with negative stiffness in controlling force transmission to a rigid foundation. *International Journal of Mechanical Sciences*, 152, 88–98. (Note: Adjusted Chinese names to proper Last Name, Initial format)
- [25] Acar, M. A., & Yilmaz, C. (2013). Design of an adaptive-passive dynamic vibration absorber composed of a string-mass system equipped with negative stiffness tension adjusting mechanism. *Journal of Sound and Vibration*, 332(2), 231–245.
- [26] Yang, J., Xiong, Y. P., & Xing, J. T. (2013). Dynamics and power flow behaviour of a nonlinear vibration isolation system with a negative stiffness mechanism. *Journal of Sound and Vibration*, 332(1), 167–183.
- [27] Zivieri, R. (2020, September 27–30). Dynamical properties of a periodic mass-spring nonlinear seismic metamaterial [Paper presentation]. Fourteenth International Congress on Artificial Materials for Novel Wave Phenomena (Metamaterials), New York, NY, United States. IEEE Xplore. <http://www.ieee.org>
- [28] Wang, Y., Li, H. X., Cheng, C., Ding, H., & Chen, L. Q. (2021). A nonlinear stiffness and nonlinear inertial vibration isolator. *Journal of Vibration and Control*, 27(11–12), 1336–1352.
- [29] Den Hartog, J. P. (1981). *Mechanical vibrations*. Dover Publications.