SOIL COUNTERACTION FORCE ACTING ON A WEDGE-SHAPED NON-AXISYMMETRIC SOIL PROCESSING WORKING ELEMENT

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Abstract - The aim of the study is to determine the static soil resistance forces acting on a non-axisymmetric wedge-shaped soil-working tool. The study examines the equilibrium state of a working element shaped like a non-axisymmetric wedge. It derives generalized formulas for the soil reaction force acting on this wedge when subjected to a constant traction force. To solve this static problem, an equilibrium equation is established by applying the principle of releasing constraints from theoretical mechanics. The resulting open system of equations is then transformed into a closed system using the dry friction formula, which effectively converts a statically indeterminate problem into a statically determinate one. The derived formulas enable the calculation of reaction forces on two wedge surfaces with different, small inclination angles - a task that could not be accomplished using methods found in the literature. The study also presents calculation results along with a comparative analysis.

Keywords: non-axisymmetric wedge, reaction forces, friction force, equilibrium equations, dry friction.

1. Introduction

Wedge-shaped working elements are widely used in soil cultivation. Energy-efficient design of such tools can be determined by calculating the reaction forces for different parameter values, such as surface inclination angle and applied traction. Calculations can be performed by programming, using ready-made software, or simple formulas understandable to specialists of all levels and derived from the well-known fundamental laws of classical mechanics.

Wedge-shaped units are commonly used in the railway industry, particularly in friction wedge brakes for large freight cars. The static positions and dynamic processes of these brakes have been extensively studied. Modern modeling methods have been employed to analyze the dynamics of friction wedge brakes, and the findings have contributed to improvements in railway car operations [1]. Another author addressed a similar issue by exploring car stability within complex train systems [2]. Wedge-shaped piles are widely used in construction. Research is being conducted for these projects to determine their effective and cost-efficient designs. Numerical modeling was conducted to determine the differences between wedge piles and piles of other shapes. The developed methodology makes it possible to determine the influencing factors when using conical, transverse, sectional, and wedge piles. Calculations for four types of piles showed that wedge piles exhibit the best performance and are one of the most cost-effective pile systems [3]

Additionally, some structures feature foundations with a wedge shape, resembling conical piles. Experimental studies have been conducted for both foundation types to assess pressure changes based on settlement, with results compared to those of other foundation types. The studies also evaluate the environmental impact of these foundations [4].

Research on the use of wedge-shaped working tools in agriculture describes the process of soil fragmentation using normal stresses that stretch the soil [5]. The authors argue that each type of soil has a specific resistance to fragmentation, displacement, or rupture under the action of a working tool. These

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problems are solved by formulating equilibrium equations for normal and tangential forces. However, stresses are expressed as a matrix, and the authors, after performing numerical calculations using the developed system of equations, concluded that the wedge differs from other working tool shapes in its fundamental properties during soil fragmentation.

A study was conducted on the interaction of a dihedral wedge with soil consisting of two layers [6]. The total resistance of this wedge is considered as the sum of the soil's sliding resistance, the gravitational force of the layer, and the dynamic pressure of the layer. Analytical relationships were also obtained to calculate the total resistance of the dihedral wedge, considering the properties of various soil layers. The results showed that conducting research using analytical expressions has an advantage over numerical modeling for this type of problem.

The results of a model of soil interaction with a dihedral wedge [7] revealed a true relationship between the sizes of soil layers and soil properties. The author of reference [8] did not limit himself to studying the interaction of the wedge with the soil, but conducted a study on the interaction of tractor tracks with the soil.

There are also studies devoted to the sliding resistance of a trihedral wedge in soil [9]. In this study, a mathematical model of the interaction of a trihedral wedge with soil was presented to the scientific community, describing the influence of digging depth and friction coefficients between the wedge and the soil on the wedge's sliding resistance. By examining the fundamental laws of force interaction between a trihedral wedge and soil [10], an analytical solution was obtained for the first time for calculating the traction resistance of a trihedral wedge along the coordinate axes. For such a wedge of various configurations, a force calculation method was developed that takes into account changes in many factors [11].

Studying the static state of wedge-shaped tools is the basis for studying their dynamics, but it does not help to reveal the nature of their motion. A significant number of studies exist on the dynamics of mechanisms with wedge-shaped working elements. A transient problem of wedge penetration into an ideal plastic semi-medium of limited width, taking into account contact friction, was investigated and solved [12]. A mathematical model of the spatial dynamic problem of a wedge, considering applied forces with different lines of action, was developed, and valuable recommendations for applying this model to practical problems were proposed [13]. In this case, the first stage of wedge penetration into an ideal plastic medium is represented by maintaining the geometric similarity of a plastic sphere.

The second stage of wedge penetration into the ideal plastic medium is modeled by the area of slip lines and the velocity hodograph. The shapes of the boundary of the half-space near the wedge and the dependence on the wedge penetration point into the half-space, up to stationary penetration into an infinitely elastic medium, are determined.

Research has been conducted on the penetration of wedge-shaped working elements into a plastic medium. The problem of wedge or cone penetration into a plastic medium was developed for a plastic medium corresponding to the double-shearing model [14]. In this model, deformation is determined by solving a system of static equations in two characteristic directions, that is, the static equations and the yield conditions. Based on this model, other authors solved the problem in polar coordinates. The problem of uniform velocity fields in the problem of wedge or cone penetration into a plastic and viscous medium is the one that attracts great interest. However, these problems were studied for the tips of the wedge or cone, where the friction law is ineffective. Solving the problem of wedge or cone entry into a plastic medium using the double-shearing model can help overcome such shortcomings.

Analytical expressions for the action and reaction forces of working tools in the soil, depending on the inclination angle and the wedge penetration rate, and methods to obtain them were analyzed in reference [15]. The authors found it difficult to use ready-made formulas for calculations, so they developed a software product for spatial problems of this type to study the patterns of change in the above parameters. The obtained numerical results were compared with measurement data; optimal values of the inclination angle and other parameters were determined; good agreement of results was obtained. The values of the parameters that ensure maximum soil porosity were also determined.

Wedge penetration into soil is an important issue. Numerous studies have been conducted in this area, including publications that take into account soil properties [16]. A mechanical-rheological model of the interaction processes between ground and soil was developed, allowing for the soil properties to be accounted for. In deriving this rheological law, the soil was considered a solid, continuous body, and a rheological law was developed.

In this article, the authors derived formulas for the reaction force of a wedge-shaped working element penetrating the soil for two working surfaces of different angles of inclination.

2. Materials and Methods

We consider the statics of a working element penetrating the soil under the action of a constant force at different inclination angles. We determine the reaction forces of this wedge (Fig. 1). In the literature [17, 18], this normal force is defined by the following formula:

$$N = P / \sin\alpha \quad (1)$$

and it has no numerical value at small inclination angles, making it impossible to determine the reaction forces of a non-axisymmetric wedge.

To solve the problem, we apply the principle of constraint release in theoretical mechanics [19], replacing constraints with reaction forces, constructing a force equilibrium equation, and finding the unknown forces from these equations. Instead of an expression for the reaction force, which becomes infinite for certain values of the denominator, we obtain a generalized expression for the reaction force that reflects the true nature of the physical process even at small inclination angles.

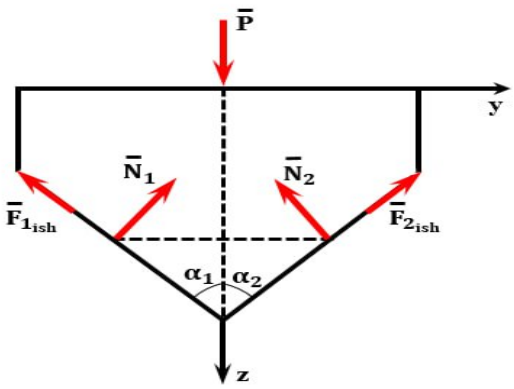

Figure 1: Active and passive forces

We set rectangular Cartesian coordinate axes (Fig. 1) and direct the *y*-axis perpendicular to the direction of the acting force, and the *z*-axis along this direction. It is known that the normal reaction forces arising on the wedge surfaces are directed perpendicular to these surfaces. We denote the normal reaction forces by $\bar{N}_1$ and $\bar{N}_2$. Naturally, frictional forces arise between the wedge surfaces and the medium into which it penetrates. Frictional forces $\bar{F}_{ish1}$ and $\bar{F}_{ish2}$ arising on each working surface between the wedge and the medium are directed in the opposite direction to the applied force along the two surfaces of the wedge.

As a result, we obtain the following equilibrium equation for the force projections onto the *y*- and *z*-axes:

$$N_1 \cos\alpha_1 - F_1 \sin\alpha_1 - N_2 \cos\alpha_2 + F_2 \sin\alpha_2 = 0; \quad (2)$$

$$-N_1 \sin\alpha_1 - F_1 \cos\alpha_1 - N_2 \sin\alpha_2 - F_2 \cos\alpha_2 + P = 0. \quad (3)$$

The number of resulting equations exceeds the number of unknowns, meaning we are dealing with a statically indeterminate problem or an open system of equations. To reduce this statically indeterminate problem to a statically determinate one, we must reduce the number of equations. To do this, we use the dry friction formula $F = fN$ (where f - is the friction coefficient) [19], which relates the normal force to the friction force, and apply equalities $F_1 = fN_1$, $F_2 = fN_2$.

First, from the equilibrium equation (2) for the projections of forces onto the *y*-axis, we obtain:

$$N_1 \cos\alpha_1 - fN_1 \sin\alpha_1 - N_2 \cos\alpha_2 + fN_2 \sin\alpha_2 = 0$$

or

$$N_1(\cos\alpha_1 - f\sin\alpha_1) - N_2(\cos\alpha_2 - f\sin\alpha_2) = 0. \quad (4)$$

From (3), we obtain:

$$-N_1(\sin\alpha_1 + f\cos\alpha_1) - N_2(\sin\alpha_2 + f\cos\alpha_2) + P = 0. \quad (5)$$

From (4), we determine the relationships between N_1 and N_2:

$$N_2 = N_1 \frac{\cos\alpha_1 - f\sin\alpha_1}{\cos\alpha_2 - f\sin\alpha_2}.$$

$$N_1 = N_2 \frac{\cos\alpha_2 - f\sin\alpha_2}{\cos\alpha_1 - f\sin\alpha_1} \quad (6)$$

Substituting this into (5), we obtain:

$$N_2 \frac{\cos\alpha_2 - f\sin\alpha_2}{\cos\alpha_1 - f\sin\alpha_1}(\sin\alpha_1 + f\cos\alpha_1) + N_2(\sin\alpha_2 + f\cos\alpha_2) - P = 0 \quad (7)$$

or

$$N_2\left[\frac{(\cos\alpha_2 - f\sin\alpha_2)(\sin\alpha_1 + f\cos\alpha_1) + (\sin\alpha_2 + f\cos\alpha_2)(\cos\alpha_1 - f\sin\alpha_1)}{\cos\alpha_1 - f\sin\alpha_1}\right] - P = 0.$$

Having determined N_2 from the latter and using (6), we finally find the moduli of the reaction forces:

$$N_2 = \frac{P(\cos\alpha_1 - f\sin\alpha_1)}{(\cos\alpha_2 - f\sin\alpha_2)(\sin\alpha_1 + f\cos\alpha_1) + (\sin\alpha_2 + f\cos\alpha_2)(\cos\alpha_1 - f\sin\alpha_1)}. \quad (8)$$

$$N_1 = \frac{P(\cos\alpha_2 - f\sin\alpha_2)}{(\cos\alpha_2 - f\sin\alpha_2)(\sin\alpha_1 + f\cos\alpha_1) + (\sin\alpha_2 + f\cos\alpha_2)(\cos\alpha_1 - f\sin\alpha_1)}. \quad (9)$$

Analytical expressions (8) and (9) are obtained, which determine the values of the normal reaction forces arising on the working surfaces of a nonaxisymmetric wedge with different inclination

angles α_1 and α_2 under the action of an applied constant force $\bar{P}$.

3. Calculation Results

Calculations were conducted for a wedge with various inclination angles using formulas (8) and (9). Figure 2 illustrates how the wedge's constraint reaction force, specifically the soil reaction force, changes with the second inclination angle while keeping the first inclination angle constant (represented by non-solid lines).

The calculations were based on the following parameters: applied force $P=100$ N; friction coefficient $f=0,2$; and specified value for the first inclination angle $\alpha_1=15^0$. The solid line in the graph indicates results obtained using formula (1), where the reaction force at a zero inclination angle approaches infinity and does not have a specific numerical value. In contrast, the non-solid lines, representing the results from formulas (8) and (9), highlight the advantages of these derived equations.

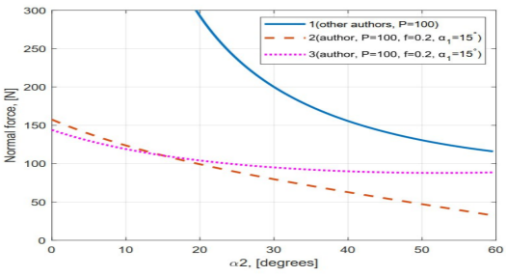

Figure 2: Wedge reaction forces for $P=100N$; $f=0,2$; $\alpha_1=15^0$

Figure 3 shows the calculation results for the following parameter values: $P=200$ N; $f=0,2$; $\alpha_1=15^0$. Here and in the previous figure, the intersection of two lines can be seen. The intersection points in the figures actually correspond to identical inclination angles α_1 and α_2. This means that, for equal angles on the two working surfaces, the reaction forces have the same values. This confirms the validity of the obtained formulas for the reaction force of a non-axisymmetric wedge.

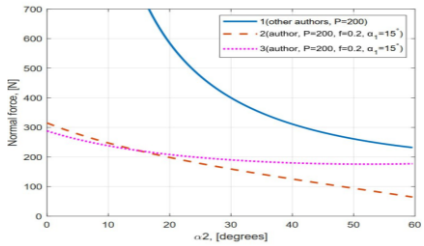

Figure 3: Wedge reaction forces for $P=200N$; $f=0,2$; $\alpha_1=15^0$

The results of calculations regarding the change in the reaction force of sandy soil based on the first angle of inclination α_1 at a constant second angle of inclination $\alpha_2=30^0$ (broken lines) are presented in Figures 4 and 5. Figure 4 illustrates this relationship for $P=100N$ and $f=0,2$. The solid line in this figure represents the result obtained from formula (1).

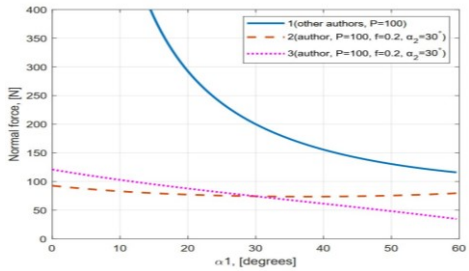

Figure 4: Constraint reaction forces of the wedge for $P=100$; $f=0,2$; $\alpha_2=30^0$

Figure 5 depicts the variations in the wedge reaction force for $P=200N$; $f=0,2$; $\alpha_2=30^0$. A comparison of Figures 4 and 5 indicates that when the applied force is doubled, the reaction force at the extreme point $\alpha_1=0^0$ also doubles.

The results obtained for the friction coefficient using formulas (8) and (9) are shown in Figures 6 - 9. In Figures 6 - 7, the first angle is $\alpha_1=15^0$, and the second angle varies. In Figures 8 and 9, the first angle ranges from 0 to 60 degrees, and the second angle is $\alpha_2=30^0$.

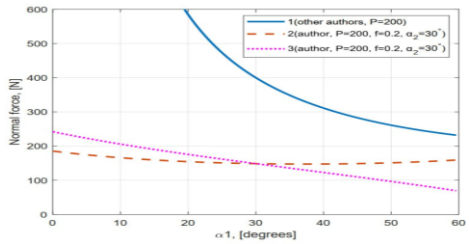

Figure 5: Constraint reaction forces of the wedge for $P=200N$; $f=0,2$; $\alpha_2=30^0$

The results for the friction coefficient, derived using formulas (8) and (9), are displayed in Figures 6 to 9. In Figures 6 and 7, the first angle is $\alpha_1=15^0$, while the second angle varies.

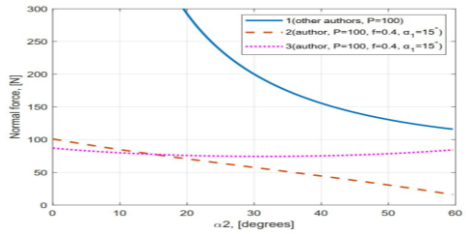

Figure 6: Constraint reaction forces of the wedge for $P=100N$; $f=0,4$; $\alpha_1=15^0$

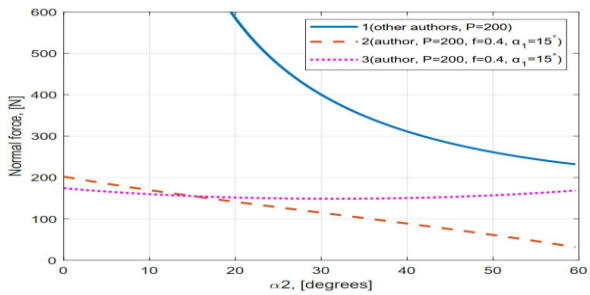

Figure 7: Constraint reaction forces of the wedge for $P = 200N$; $f = 0{,}4$; $\alpha_1 = 15^0$

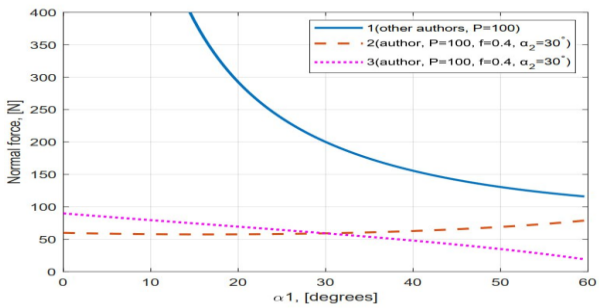

Figure 8: Constraint reaction forces of the wedge for $P = 100N$; $f = 0{,}4$; $\alpha_2 = 30^0$

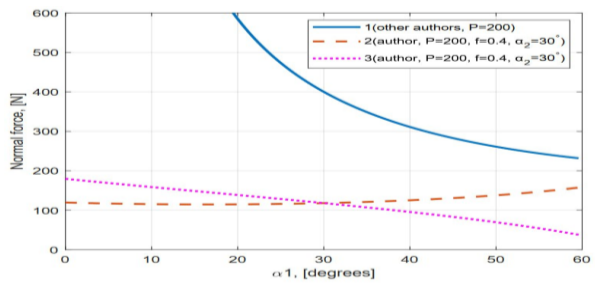

Figure 9: Constraint reaction forces of the wedge for $P = 200N$; $f = 0{,}2$; $\alpha_2 = 30^0$

According to formulas proposed by other researchers, the reaction force approaches infinity as the angle of inclination nears zero. However, our findings, based on formulas (8) and (9), show that the reaction forces converge to specific numerical values.

Calculations were also performed for other values of the basic parameters, the graphs of which are similar to those in Figures 2 and 3, differing only in their numerical values.

4. Conclusions

Generalized formulas have been developed to determine the soil reaction force acting on a wedge-shaped working element at various inclination angles under the influence of a constant force. Using analytical expressions derived from classical laws and methods of mechanics is much easier than developing a software product based on various theories when studying the state of this type of object. Given identical angles of inclination, the solutions previously derived and published by the authors for other wedge geometries can be easily obtained from the final formulas.

These formulas provide specific numerical values for the normal reaction forces at any wedge inclination angle, including challenging small angles. This highlights the reliability and advantages of the results presented in this work. However, it is important to note that calculations for small angles of inclination can still be inaccurate when dealing with wedge problems, both in rigid-body theory (theoretical mechanics) and in its practical applications.

The derived formulas can be applied in several areas, including the calculation of wedge valves for oil pipeline transport, wedge spring-friction assemblies for freight car bogies, V-belt and wedge friction drives for agricultural machinery, piles in construction, wedge-shaped soil cultivation elements, and numerous other wedge-shaped components in various industries during their design process. The findings can also be included in technical literature as fundamental formulas for analyzing the action and reaction of wedge-shaped technical objects. Furthermore, the method of converting statically indeterminate problems into statically determinate problems can be applied to address other practical issues.

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